THE UNIVERSITY OF ALBERTA

AN INVESTIGATION OF THREE MATHEMATICS HALF-COURSES CONDUCTED AT THE GRADE TWELVE LEVEL

by

© DANIEL JOSEPH TESSARI

A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled An Investigation of Three Mathematics Half-Courses Conducted at the Grade Twelve Level submitted by Daniel Joseph Tessari in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of this study was to evaluate three mathematics half-courses taught as part of the Mathematics 31X course during the 1966-67 school year in Alberta. These half-courses were Calculus, Probability and Statistics, Matrix Algebra. Four groups of students were used in the study; three experimental groups and one control group. The students in the control group studied the normal Mathematics 31 course (plane trigonometry and related topics) during the 1966-67 school year. On the basis of SCAT scores, each group was divided into three ability levels which were labelled moderate, high, and very high. All of the students involved in the study were enrolled in grade twelve and ranged in age from sixteen to twenty-one years.

Information was gathered from a number of sources to make the evaluation as broad as possible but still manageable. Measures of attitude, anxiety, achievement, and scholastic ability were obtained for each student. Additional information was supplied by the teachers who participated in the Mathematics 31X program and by professors at the University of Alberta. Using a number of statistical procedures, mainly analysis of variance and covariance, the investigator tested seven major hypotheses. The .05 level of significance was used in this study.

The students who studied matrix algebra showed a

significantly more favorable attitude towards mathematics than the students from the other three groups. The students in the calculus group showed a significantly higher level of anxiety than the students in the other experimental groups. A significant positive relationship existed between achievement in trigonometry and achievement in each of the half-courses. Within each of the half-courses, students of very high ability achieved significantly better than students of moderate ability on the respective half-course final test. Other significant differences in achievement were also found among the ability levels within the matrices group.

As compared to the other two half-courses, university professors selected a knowledge of calculus as being the most beneficial to the university freshman regardless of the students' major area of interest. However, there were indications that these major areas of interest should be considered separately.

The participating teachers reacted favorably to the experiment and most of them indicated that they preferred splitting the Mathematics 31 course into two parts. Only one of the fourteen teachers wanted a return to the full year of trigonometry. The teacher responses seemed to indicate that a half-course in calculus or matrix algebra is more suitable for the Mathematics 31 course than is a half-course in probability and statistics for Alberta students.

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NATURE OF THE PROBLEM

I. INTRODUCTION

For approximately fifteen years, mathematics educators across North America have been involved in a vigorous drive for quality in the organization and instruction of school mathematics. This movement, carrying a revolutionary force, has often been referred to as "modern mathematics."

Numerous experimental programs that were designed to capture the spirit of modern mathematics have been developed. Most of these programs stress the learning of mathematical concepts and principles rather than computational skills. More emphasis is placed upon reasoning, thinking, and understanding than was previously thought necessary. Greater stress has also been placed upon the development of materials that are enjoyable and challenging to most students. Finally, experimental courses stress self-motivation and promote the idea that the structure of mathematics is based on logic rather than social application.[1]

Many changes have also taken place with regard to the

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[1]Eleanor Chastain, "Objectives of Experimental Courses in Elementary Mathematics," School Science and Mathematics, 65:49-55, January, 1965.

content found in the mathematics curriculum. The social-utility mathematics which was emphasized in the traditional mathematics curriculum has all but disappeared at the junior high school level. Similarly, the emphasis upon trigonometric and logarithmic calculation has been greatly reduced in the high school courses. Another noticeable change at the high school level is the trend towards integrated courses. Topics such as plane and solid geometry along with trigonometry are often combined with algebraic topics rather than treated separately.

These changes in content have allowed curriculum makers to introduce advanced topics to students at an earlier age. This has led to extensive experimentation with college-level mathematics courses at the grade twelve level. One such program, which was undertaken in the Province of Alberta, led to the development of this study.

During the 1966-67 school year, three experimental Mathematics 31 courses were conducted in the Province of Alberta under the direction of the Senior High School Mathematics Subcommittee. To distinguish these courses from the usual Mathematics 31 course (which consists of a full year of plane trigonometry and related topics), these experimental courses were labelled Mathematics 31X. The students in each of the three experimental courses studied plane trigonometry for the first five months of the school year. During the

last five months of the school year, one group of students studied calculus, another group studied sets, probability, and hypothesis testing, while the third group studied matrix algebra. This was the first time that these courses were offered at the high school level in Alberta. The total Mathematics 31X program involved fourteen teachers, fourteen different classes, and approximately three hundred students from various cities, towns, and villages in the Province of Alberta.

II. PURPOSE OF THE STUDY

The purpose of this study was to evaluate three different half-courses taught as part of the Mathematics 31X course. These half-courses were: (1) Calculus, (2) Sets, Probability, and Hypothesis Testing, and (3) Matrix Algebra.

This evaluation was based upon the analysis of information which was obtained from a number of sources. Direct measures of attitude, anxiety, achievement, and scholastic ability were obtained for each student who participated in the study. Additional information was supplied by the teachers who participated in the Mathematics 31X program and by professors from the University of Alberta.

III. NEED FOR THE STUDY

The movement for curricular change in school math-

ematics has grown so that it can no longer be considered merely novel or transitory. There is a substantial and clear emphasis on the improvement of quality, and the impact is particularly noticeable in the advanced mathematics courses offered in many United States high schools.

What mathematics should we offer for college-bound high school seniors? Related literature shows that there is a definite lack of agreement on the mathematics that should be taught in the grade twelve courses for college-bound students. All three of the above-mentioned half-courses have been offered in various high schools in the United States. However, the extent to which they are offered varies considerably. Woodby writes:

> There is little acceptance of a course in probability and statistics as the fourth- or fifth-year mathematics offering in the college-preparatory program. There is even less acceptance of courses in linear algebra, matrices, and computer mathematics. This situation is probably due to the lack of knowledge of these subjects by the great majority of high school teachers.
>
> Most of the advanced courses are either calculus and analytic geometry, or algebra and analysis courses intended to prepare students for calculus. Although many high schools are teaching calculus courses of good quality, there is a trend toward the teaching of nonrigorous courses in calculus that emphasize only the mechanics of differentiation and integration. In effect these are warm-up courses to give students a "running start" in the beginning course in college.[2]

In stating his recommendations, Woodby claims:

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[2]Lauren G. Woodby, Emerging Twelfth-Grade Mathematics Programs, U.S. Department of Health, Education, and Welfare, (Washington: Government Printing Office, 1965), p. 35.

> Courses in calculus and analytic geometry of the warm-up variety are not recommended. If calculus is to be taught in the high school, it should be a full-year course comparable in quality to the college level course currently taught in most colleges and universities.
> Twelfth-grade courses in probability and statistics, linear algebra, or analytic geometry should be considered in preference to calculus for many high school students.[3]

It should be noted that the preceding remarks are based on the analysis of information received from sixty-six high schools in the United States by means of correspondence, classroom visits, and interviews with teachers and administrators. Many of these high schools employ acceleration and enrichment to develop strong mathematics programs and most of the students in these programs possess high ability.

The question arises as to what program is most suitable for the grade twelve level in the high schools of Alberta. Very few Alberta high schools offer comprehensive enrichment or accelerated mathematics programs. Since most students enrolled in Mathematics 31 possess above average ability, perhaps these students can benefit by studying some of these more advanced courses.

This study will attempt to indicate whether the proposed three half-courses are suitable for Mathematics 31 students, and which of the three is the most suitable. Studies of this nature are indeed lacking and research of this type would be helpful in determining appropriate content

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[3] *Ibid.*, p. 36.

for the college-bound student. On a long range basis, the results of this study may also be useful for a revision of the entire high school mathematics curriculum.

IV. CLARIFICATION OF TERMS

To assist the reader throughout this report, the following terms are defined. Other terms are defined as they arise.

Calculus group refers to those students who studied calculus during the last half of the Mathematics 31X course.

Matrices group refers to those students who studied matrix algebra (also denoted as linear algebra) during the last half of the Mathematics 31X course.

Probability group refers to those students who studied sets, probability, and hypothesis testing during the last half of the Mathematics 31X course.

Experimental groups refer to the calculus, matrices, and probability groups.

Control group refers to those students who studied the regular Mathematics 31 course for the entire year.

Half-courses refer to the calculus, matrix algebra, and probability and statistics material studied by the experimental groups.

SCAT scores refer to the total raw scores that the students obtained on the Cooperative School and College

Ability Test, Level 1B.

<u>Very high ability group</u> refers to those students who obtained a SCAT score of 91 or more.

<u>High ability group</u> refers to those students who obtained a SCAT score of 81 to 90 inclusive.

<u>Moderate ability group</u> refers to those students who obtained a SCAT score of 80 or less.

<u>Attitude scores</u> refer to the total raw score that the students obtained on the attitude items found in the attitude-anxiety questionnaire.

<u>Anxiety scores</u> refer to the total raw score that the students obtained on the anxiety items found in the attitude-anxiety questionnaire.

<u>Achievement scores</u> refer to the total raw score that the students in the experimental groups obtained on their respective half-course final test.

V. DELIMITATIONS

Although the study was intended to evaluate the three experimental half-courses as fully as possible, the investigator found it necessary to delimit the study as follows:

1. No attempt was made to assess the influence that the participating teachers may have exerted upon the students' attitude, anxiety, and achievement scores.
2. No attempt was made to control home, school, or community

influences that may have affected this study.

3. No attempt was made to evaluate these half-courses using one specific curriculum development theory. There are two reasons why this was not done. Firstly, there seems to be no one theory of curriculum development that is accepted by the majority of educators. Secondly, if such a theory did exist, it would be so vast and unwieldy that for practical purposes it could not be tested in its entirety.
4. The only mathematical objective considered in this study is the general objective of Mathematics 31--namely, to provide college-bound students with suitable mathematical experiences that will encourage them to continue their study of mathematics and/or related fields at university.

VI. PREVIEW OF THE THESIS

The present chapter has dealt with the nature of the problem. Chapter II consists of a review of literature related to this study while Chapter III includes a detailed description of the theory and design of the study. The results from the statistical analyses are presented in Chapter IV and Chapter V contains a summary of the investigation, limitations, recommendations and implications for further research.

CHAPTER II

REVIEW OF RELATED LITERATURE

I. INTRODUCTION

In this chapter a review of some of the literature pertinent to this study is reported. For the convenience of the reader, this review is divided into three parts. The first portion of this chapter is devoted to a summary of some of the extensive curriculum projects that have been undertaken to update mathematics programs at the secondary level. Emphasis is placed upon that content suggested in these projects, especially content considered suitable at the grade twelve level. The views of some mathematics educators concerning the placement of topics in the grade twelve course is also given.

Part two contains a review of curriculum studies that are directly concerned with the three half-courses being evaluated in this study. Some of these studies are experimental in nature while others are survey studies.

The remaining part of the chapter contains a review of certain studies which analyze factors related to the learning of mathematics. Since the present study is concerned with attitude, anxiety, and ability, only these factors will be discussed.

II. THE COLLEGE PREPARATORY MATHEMATICS CURRICULUM

Secondary School Mathematics Projects

Until recently, the mathematics curriculum in elementary and secondary schools has been relatively unchanged for a century or more. For the most part, the sequence of topics and their grade placement was determined by tradition rather than by efforts to discover what could be learned most effectively by children at various age levels. Mathematical principles and concepts developed in the last half century were treated briefly, or not at all, in the school curriculum. The increasing need on the part of scientists for competence in the "new" mathematics as well as increased recognition of the subject's intrinsic importance has given a double impetus to curriculum studies in this field.

Recommendations for college-preparatory mathematics in grades nine through twelve were published by the Commission on Mathematics of the College Entrance Examination Board in 1959, following a three year study. The Commission called for a revised high school program permitting graduates to begin their college mathematics at the level of calculus and analytical geometry. The following programs were outlined by the Commission:

> Grade 9 - Elementary Mathematics I. The theme of this course would be the nature and use of variables, with the elementary ideas and notions of "sets" employed to sim-

plify, clarify, and unify the introduction to algebra. At the same time, the students would work with both inequalities and equations, and the properties of the number system would be kept to the fore at all times.
Grade 10 - Elementary Mathematics II. In this year the theme would be geometry and deductive reasoning. Some coordinate geometry and the essentials of solid geometry and space perception would be incorporated with somewhat curtailed treatment of traditional plane geometry.
Grade 11 - Intermediate Mathematics. This year would include algebra and elementary trigonometry centered around coordinates, vectors, and complex numbers. The theme would be real and complex numbers.
Grade 12 - The Commission offers three suggested programs for grade twelve. In the first two of these, a core course on Elementary Functions would be taught during the first semester; Introductory Probability would complete the year in the one program and Introduction to Modern Algebra in the other.[1]

Whereas the CEEB Commission of Mathematics restricted its attention to the college-capable student, the Secondary School Curriculum Committee of the National Council of Teachers of Mathematics examined mathematics programs for all students in the secondary school. It also reviewed current trends and proposals in teaching mathematics in the high schools.

The committee called for readjustment of emphasis in both objectives and course content. Basic concepts and principles would receive increased emphasis, with explicit attention devoted to the structure of mathematics and to certain principles of logic. On the other hand, emphasis upon meaningless memorization and drill was to be diminished.

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[1] Commission on Mathematics, A Summary of the Report of the Commission on Mathematics (Princeton: College Entrance Examination Board, 1959), pp. 8-9.

Every student, urged the committee, should have the opportunity to study four years of high school mathematics (grade nine through twelve) in a program appropriate to his abilities and interests. Two years of mathematics should be required of all students for high school graduation. For college-preparatory students, the following curriculum was recommended;

> Grade 9. Algebra
> Grade 10. Geometry--chiefly plane, some solid
> Grade 11. Algebra and Trigonometry
> Grade 12. Two one-semester courses selected from probability and statistics, analytic geometry, mathematical analysis based on a study of functions. For Advanced Placement examinations, analytic geometry and calculus.[2]

In the ten years since the University of Illinois Committee on School Mathematics began its work, it has prepared text materials for a new college preparatory mathematics curriculum, grades nine through twelve. These text materials were given extensive classroom trials, revised on the basis of the results achieved, and are now available in eight units. The content and sequence of the new curriculum are indicated by the unit titles:

> Grade 9: The arithmetic of the real numbers: generalizations and algebraic manipulation; equations and inequations; applications; ordered pairs and graphs.
> Grade 10: Relations and functions: geometry.
> Grade 11: Mathematical Induction: sequences.
> Grade 12: Exponential and logarithmic functions; circular functions and trigonometry; polynomial func-

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[2]National Council of Teachers of Mathematics, Secondary School Curriculum Committee, "The Secondary Mathematics Curriculum," The Mathematics Teacher, 52:402, May, 1959.

tions and complex numbers.[3]

As the UICSM project has developed, the text materials have been designed and teaching methods suggested to stress discovery of generalizations by students as opposed to presenting the generalizations and then explaining them. The Committee stresses the sequential nature of its four year program and warns against attempting to use parts of it to supplement a program that is organized on a different basis.

The secondary school mathematics program of the School Mathematics Study Group, like the recommendations of the CEEB Commission on Mathematics, emphasizes the introduction of modern mathematics and a deeper treatment of traditional mathematical topics within the course structure generally found in United States high schools, rather than a radically different sequence of courses.

The proposed syllabus for high school is very similar to that suggested by the CEEB Committee. However, in the twelfth year, the SMSG emphasizes the following:

> Elementary functions, developed in a manner to prepare for calculus and to convey the "spirit of mathematical thinking", an introduction to matrix algebra, including research exercises that may be used to introduce able students to mathematical research.[4]

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[3]Dorothy M. Fraser, Current Curriculum Studies in Academic Subjects (Washington: National Education Association, 1962), pp. 30-31.

[4]*Ibid.*, pp. 33-34.

One quickly realizes that the four programs outlined in the previous paragraphs are not radically different in their objectives. All of these programs call for a comprehensive, four-year, high school program to prepare the college-bound student for further mathematical study. However, each project allows for some latitude in the choice of topics for grade twelve.

During the summer of 1963, a group of twenty-nine professional mathematicians and natural scientists met to review school mathematics and to discuss mathematical education. The major areas of discussion centered around the broad goals of the school mathematics curriculum, pedagogical principles and techniques, and the development of a desirable mathematics curriculum from kindergarten through grade twelve. This conference became known as the Cambridge Conference.[5]

Although the members of this conference recognized the successes of such programs as those developed by the School Mathematics Study Group and the University of Illinois Curriculum Study in Mathematics, they also pointed out some short comings. One major factor that is limiting the effectiveness of these new programs is the scarcity of adequately

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[5] Educational Services Incorporated, <u>Goals for School Mathematics</u> (The Report of the Cambridge Conference on School Mathematics. Boston: Houghton Mifflin Company, 1963).

trained teachers who are capable of leading the students to the level of mathematics set down within these programs. Many of these programs are therefore forced to operate within the basic framework of the classical curriculum. Hence these reform curricula may become frozen into the educational system and set an upper limit on the student's knowledge of mathematics which will be inadequate in the future.

The conference outlined a bold mathematics curriculum that demonstrated its impatience with the present capacities of the educational system. The content of this curriculum may be summarized as follows:

> The subject matter which we are proposing can be roughly described by saying that a student who has worked through the full thirteen years of mathematics in grades K to 12 should have a level of training comparable to three years of top-level college training today; that is, we shall expect him to have the equivalent of two years of calculus, and one semester each of modern algebra and probability theory. At first glance this seems to be totally unrealistic; yet we must remember that, since the beginning of the century, there has been about a three-year speed-up in the teaching of mathematics....[6]

Within this curriculum, the students would meet the same topic at several different stages. For example, a student may study topics in probability in every grade from seven through twelve.

Current Views on Grade Twelve Mathematics

The preceding section outlined the four-year high

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[6] *Ibid*., p. 7.

school mathematics program recommended for the college-bound mathematics student. One area of concern for curriculum planners has been finding suitable mathematical material for grade twelve students, especially for the second semester. There appears to be considerable controversy over what should be taught.

One course that is often taught at the grade twelve level is calculus. Beninati claims that the popularity of teaching calculus at the high school level is attributable to the lack of properly prepared young people to fill the ranks of students in technological areas where critical shortages exist. He goes on to say:

> A crash course to teach calculus in response to various pressures may result in a course's being taught by inadequately prepared teachers to less than superior pupils. The course could become a meaningless mechanical manipulation of symbols that results in a less than adequate preparation in an area of mathematics so important to the understanding of more advanced work.[7]

He suggests a course in probability and statistics would give the students a chance to mature mathematically while studying an area of mathematics that has increasing significance in the social and behavioural sciences.

Allendoerfer also takes a firm stand against teaching

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[7] A. Beninati, "It's Time to Take a Closer Look at High School Calculus," The Mathematics Teacher, 59:29, January, 1966.

calculus in the high school. He writes:

> As I understand the situation, the argument for early calculus is that calculus is so overwhelmingly important that it should displace any other subject in the curriculum. But for what is it so important? The physicists, chemists, and engineers will tell us that their students need it in the beginning courses in these subjects. If you press them, however, these scientific friends will tell you that even more important than calculus is a knowledge of algebra, trigonometry, and geometry....
>
> I therefore submit that an essential prerequisite for calculus is a full treatment of analytic geometry. Indeed I urge no school teach any calculus, probability, matrix algebra, or finite mathematics until this analytic geometry has been well-presented to the students.[8]

Allendoerfer's strong case for analytic geometry stems from two reasons. Firstly, colleges have largely abandoned it, so that if it is to be taught at all, it must appear in the high school. Secondly, most high school teachers could teach it without special training.

Other writers expound different views. Blank feels that the preparation for calculus could be accomplished by the end of the third year of senior high school. For those students who select another year of mathematics in grade twelve, the course should be calculus and a full year of it. He defends his position as follows:

> Why not something else? Why not probability and statistics, or an introduction to modern algebra, or a semester on elementary functions followed by a semester of linear algebra? Stripped of detail my answer is

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[8] C. B. Allendoerfer, "The Case Against Calculus," The Mathematics Teacher, 56:483-484, November, 1963.

> simply that the calculus provides the broadest possible base of experience for the student's later development which we seek.[9]

Williams definitely agrees that probability and statistics should be taught either at the grade twelve level or as a freshman course at the junior college.[10] Again he cites the impact of the course upon society and the fact that it has been taught successfully at the grade twelve level. However, he feels that it is the responsibility of the mathematics department of each high school to decide whether or not the course should be included in the syllabus.

Another writer, Grossman, agrees with the value of probability and statistics as a subject for general education but does not agree that it should be taught to the mathematically-gifted student in the twelfth grade.[11] He suggests that other topics such as analytic geometry and calculus, nature of number systems, abstract mathematical systems, theory of limits, and perhaps some linear algebra are more

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[9] A. A. Blank, "Remarks on the Teaching of Calculus in the Secondary School," <u>The Mathematics Teacher</u>, 50:538, November, 1960.

[10] H. E. Williams, "The Place of Probability and Statistics in the Mathematics Curriculum of the Senior High School and Junior College," <u>The Mathematics Teacher</u>, 54:316-320, May, 1961.

[11] G. Grossman, "Probability and Statistics in the Twelfth Year?" <u>The Mathematics Teacher</u>, 54:540-546, November, 1961.

important for these students. If calculus and analytic geometry are taught, it should be restricted to the upper three per cent of the students whom he feels are capable of handling the Mathematics Advanced Placement program.

The views presented on the preceding pages by no means exhaust all of the literature concerning what should be taught at the grade twelve level. However, a summary of these views lead to the following conclusions:

1. None of these college level courses should be attempted unless qualified teachers are available and the students have completed the usual high school mathematics curriculum.
2. Both analytic geometry and probability and statistics are recommended for grade twelve advanced topics. Other topics such as linear algebra, modern algebra, and vector analysis also seem appropriate.
3. Whether or not calculus should be taught in the high school is questionable. More information on this topic is presented in part three of this chapter.

III. STUDIES CONCERNED WITH MATHEMATICS CURRICULA

Experimental Studies

Bridges conducted an experiment to determine the ability of apt secondary school students to apply principles

of elementary probability and statistical inference following a semester of instruction in this area.[12] During this semester, the students studied such topics as components of a statistical investigation, measures of central tendency, elementary probability, types of distributions, and hypothesis testing.

A test of general proficiency in elementary statistical inference was administered to a control group and to an experimental group before the experiment started. The testing instrument was found to be valid and reliable. The control group received no instruction whatsoever, while the experimental group received eighty hours of instruction in the areas mentioned above. At the end of this time, Bridges administered a parallel-form test to both groups and found that the experimental group performed significantly better. There were no significant differences in the scores obtained by the two groups on the initial test.

Olander also prepared an experimental unit on statistics and taught it to a class of high school sophomores of high ability.[13] He reported that the I.Q. range of the

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[12] Charles M. Jr. Bridges, "Application of Elementary Statistics in Analysis of Data by Selected Secondary School Students," Dissertation Abstracts, 20:1223.

[13] Clarence Olander, "Let's Teach Statistics," The Mathematics Teacher, 51:253-260, April, 1958.

students involved was 119-136 with the median being 125. His experiment turned out successfully and he found that these students could apply statistical concepts.

A study was undertaken to determine the effects of high school calculus on students' first-semester calculus grades at the University of Virginia.[14] Of the entering first-year men at this institution in the 1963-64 school year, eighty-three had taken one or more semesters of calculus in high school but did not obtain advanced placement in calculus. They were required to take one or more semesters of mathematics at the University of Virginia, one of which included a first semester of calculus.

In order to investigate the effects of high school calculus on their performance in the first semester of college calculus, the grades they would have been expected to earn had they not studied calculus in high school were predicted. This prediction was obtained by using regression equations based on the data from the 1963-64 entering first-year men who had taken essentially no high school calculus. Three regression equations were calculated, one for each of the three different mathematics sequences available at the

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[14] William D. McKillip, "The Effects of High School Calculus on Students' First Semester Calculus Grades at the University of Virginia," The Mathematics Teacher, 59:470-472, May, 1961.

University of Virginia.

Two principal findings came out of the study. The grades of subjects who had taken one semester but less than two complete semesters of calculus in high school were not significantly different from the grades they would have been expected to earn had they not taken calculus in high school. However, those subjects who had taken two or more semesters of high school calculus achieved significantly better grades than they would have been expected to earn had they not taken calculus in high school.

Another study was undertaken to determine, which, if either of the curricula, SMSG (which includes matrix algebra) or conventional, provided the better preparation for a first course in calculus.[15] The subjects in the study were members of a 1962-63 freshman class. The SMSG group was composed of students who had graduated from high school in 1962 with a background in SMSG materials.

The control group was also composed of 1962 high school graduates with no SMSG background. The criteria for success in calculus was established as the final grade on a calculus test devised by Coon.

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[15] Lewis Coon, "SMSG Mathematics as a Factor Influencing Success in Freshman Calculus" (unpublished Doctoral Dissertation, Oklahoma State University, Oklahoma City, 1963).

The data were analyzed using the analysis of co-variance method to adjust for initial differences in ability and achievement. Post-test mean scores indicated a significant difference at the .01 level favoring the SMSG group in achievement in calculus.

Wick found no significant differences in the quality of preparation for first-year college mathematics between the experimental (SMSG) and the traditional mathematics curricula.[16] He also found that the student's high school mathematics record was consistently the best predictor of success in first-year college mathematics.

Opinion Surveys

Leissa and Fisher conducted a survey of teachers' opinions of the recommendations set down by the Commission on Mathematics of the College Entrance Examination Board Report entitled "Program for College Preparatory Mathematics."[17] In 1959, approximately 280 persons, primarily high school and college mathematics teachers from throughout Ohio, attended a symposium to discuss the material in the report mentioned above. All of the participants were asked to fill

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[16] Marshall E. Wick, "A Study of the Factors Associated With Achievement in First-Year College Mathematics," (unpublished Doctoral Dissertation, University of Minnesota, Minneapolis, 1963).

[17] Arthur W. Leissa and Robert C. Fisher, "A Survey

out a questionnaire containing fifteen questions formulated directly from statements found in the report. For thirteen of these questions, a response of yes or no was requested.

In general, most of the respondents reacted favorably to the recommendations set down in the report. Eighty-seven per cent of the teachers agreed that extensive revisions of the present traditional high school mathematics courses are necessary. Ninety-seven per cent of the respondents agreed that calculus is primarily a college subject, and that only school with exceptional staffs and exceptional students should offer it. Other reported percentages were: 80% in favor of probability; 65% favored determinants; 8% selected vector analysis; 8% selected differential calculus; 5% indicated integral calculus. Another interesting finding was that the college teachers were less concerned than were the high school teachers about introducing new topics into the high school mathematics curriculum.

These results were similar to what Brown found for a similar sample.[18] In this survey, Brown was concerned with

of Teachers'Opinions of a Revised Mathematics Curriculum," The Mathematics Teacher, 53:113-118, February, 1960.

[18]Robert S. Brown, "Survey of Ohio College Opinions with Reference to High School Mathematics Programs," The Mathematics Teacher, 56:245-247, April, 1963.

determining the content most suitable for the second semester in the grade twelve course. Thirty-two completed questionnaires from Mathematics teachers in various colleges and universities dealt with this topic.

Over half of the questionnaires indicated that analytic geometry was the popular choice for the second semester. Sixty-nine per cent indicated that they did not favor teaching calculus for various reasons. A unit on probability was ranked fifth while a unit on statistics was ranked seventh. A semester dealing with matrix algebra and determinants placed sixth with calculus in the tenth and last position.

Buchanan's results also indicated what Brown had found.[19] However, Buchanan reported that many college mathematics teachers favored a unit on limits for the second semester in grade twelve.

Blank sent out questionnaires to administrators and mathematics teachers involved in an accelerated program in mathematics for students of superior ability in grades nine

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[19] O. L. Buchanan, Jr., "Opinions of College Teachers of Mathematics Regarding Content of the Twelfth-Year Course in Mathematics," The Mathematics Teacher, 58;223-225, March, 1965.

through twelve.[20] Reactions to this program were favorable with both teachers and administrators indicating that the program contained good quality mathematics that created more student interest than did the traditional curriculum. Studies by Fredstrom and Lang also indicate that accelerated mathematics programs can be successful within limits and should be continued.[21,22]

The foregoing two sections indicate some of the research related to mathematics curricula that has been done. There are many other comparative studies between different mathematics curricula that are also reported. However, these bear little relationship to this study. A good summary of the research done in the field of the secondary school mathematics is given by Brown and Abell:

> Many of the studies at the high school level are directly related to the so-called "new" mathematics. Considerable interest was shown in the evaluation of the SMSG material. Most of these evaluation studies

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[20] William R. Blank, "A Survey Concerning Advanced Mathematics Curriculum," <u>The Mathematics Teacher</u>, 57:208-211, April, 1964.

[21] Paul N. Fredstrom, "An Evaluation of the Accelerated Mathematics Program in the Lincoln, Nebraska Public Schools" (unpublished Doctoral Dissertation, University of Nebraska Teachers College, Lincoln, 1964.

[22] Robert W. Lang, "A Study of an Accelerated Mathematics Program" (unpublished Doctoral Dissertation, Wayne State University, Detroit, 1962).

compared, by means of traditional tests, the achievement of pupils who studied traditional material with those who studied SMSG material. The traditional tests showed that the pupils in the new programs did learn traditional material. There is evidence that pupils in the new program learned material that the other pupils did not. The research gives little information on the effect of the new programs in developing the pupil's ability either as a scientist or non-scientist.

Research shows that high school pupils can learn many of the concepts traditionally reserved for college students. Many mathematicians enthusiastically recommend that the "new" topics be introduced into the high school program. Others strongly discount the value of these new topics for high school youth. Research gives little or no information as to their value. Decisions on the introduction of these programs in general is based on the opinions of educators and/or mathematicians.[23]

IV. FACTORS RELATED TO THE LEARNING OF MATHEMATICS

Ability

Factor analytic studies into mathematical ability have shown that there appear to be two main types of factors which account for the greater part of variance in the obtained test scores: these are the cognitive and the affective or emotional factors. In 1951, Barakat studied the cognitive factors associated with the progress of grammar school pupils in elementary mathematics.[24] The subjects

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[23]Kenneth F. Brown and Theodore L. Abell, "Research in the Teaching of High School Mathematics," The Mathematics Teacher, 59:53-37, January, 1966.

[24]M. K. Barakat, "Factors Underlying the Mathematical Abilities of Grammar School Pupils," British Journal of Educational Psychology, 21:239-240, 1951.

involved were 160 boys and 160 girls ranging in age from thirteen to fifteen years. Using a factorial approach, Barakat found that the most significant factor was "innate intelligence". Evidence also showed that specialized factors or groups of factors such as mathematical reasoning may also be present.

Hungerman designed a comparative study to measure the variables of intelligence, socio-economic background, conventional arithmetic achievement, contemporary mathematics achievement, and attitude towards mathematics for a group of pupils who had studied contemporary mathematics against similar data for a control group who had no such experience.[25] The subjects in the experimental group were sixth-graders who had studied the SMSG program for grades four, five, and six. The subjects in the control group were also sixth-graders who had studied conventional mathematics during the same grades. Both groups were given a conventional mathematics test and a contemporary mathematics test. Achievement, both conventional and contemporary, demonstrated a marked positive relationship to intelligence. The experimental group correlation for I.Q. and conventional achieve-

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[25] Ann D. Hungerman, "Achievement and Attitude of Sixth Grade Pupils in Conventional and Contemporary Mathematics Programs," The Arithmetic Teacher, 14:30-39, January, 1967.

ment was .71 while the correlation for I.Q. and contemporary achievement was .68. This indicates that performance was related to ability to the same degree for both programs.

Johnson also conducted a study that was designed to determine both the interaction of a number of variables and the unique contribution of each to mathematics achievement.[26] The sample for the experiment consisted of 3,336 English students representing fifty-eight schools in and around London and 1,922 American students representing schools in the Minneapolis area. All students wrote the Raven Progressive Matrices Test and on the basis of these scores the classes were segmented into groups of high, medium, and low intelligence with I.Q. cutting points roughly at 117 and 100. Of all the variables studied, intelligence as measured by the Raven test proved to be the variable most related to achievement. The extent of the relationship varied according to the ability level and the country in question.

The evidence presented above also agreed with Wrigley's finding that the one cognitive variable most predictive of success in mathematics is "g" or general intelligence.[27]

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[26] Sonia A. Johnson, "Some Selected Classroom Variables and Their Relationship to Mathematics Achievement in Central Minnesota and the Greater London Area," Dissertation Abstracts, 27:139-A, No. 1.

[27] J. Wrigley, "The Factorial Nature of Ability in

Lewis also found that a factor of general education ability was found to enter into all school subjects that he tested and that verbal, manual, and arithmetic group factors were also suggested.[28]

Despite the many findings that indicate the marked positive relationship between academic achievement and ability, this relationship may well depend upon the sample tested. For example, Gibbons and Savage used a sample of sixty students at a teacher training college and found that for this limited sample, there was no significant relationship between the intellectual level of the student and academic success.[29]

This leaves the affective aspects of mathematics ability, or the emotional component as some investigators call it. This may also be complex and made up of different kinds of factors. On the one hand, it seems that certain types of personality are better suited to mathematical study than others: emotional instability and anxiety have been shown to affect adversely performance in mathematics, as that subject is

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Elementary Mathematics," British Journal of Educational Psychology, 28:61-78, 1958.

[28] D. G. Lewis, "Group Factors of Attainment in Grammar School Subjects," British Journal of Educational Psychology, 31:241-248, June, 1961.

[29] K.C. Gibbons and R.D. Savage, "Intelligence, Study Habits, and Personality Factors in Academic Success--A Preliminary Reoprt." The Durham Research Review, 5:8-12, September, 1965.

presently taught, to a relatively greater extent than performance in other subjects. This aspect may be called a primary factor of emotionality in mathematics, as opposed to the secondary or derived factors which are specific to mathematics and which are not necessarily part of the basic personality pattern. These secondary factors are, of course, the attitudes displayed by children and adults towards arithmetic and mathematics--the negative ones varying from a simple distaste to what amounts to a veritable phobia.

Anxiety

Let us turn now to this primary factor of emotionality in mathematics and to anxiety in particular. Many studies have been conducted to illustrate the effect of anxiety on academic performance in general. McCandless and Castaneda administered the children's form of the Manifest Anxiety Scale to a large school population and reported correlations between it and various aspects of school achievement.[30] They concluded that there is a tendency for the more complicated skills (reading, arithmetic, complex concept formation) to suffer more interference from anxiety than the mnemonic skills such as spelling. The correlations between

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[30] B. McCandless and A. Castaneda, "Anxiety in Children, School Achievement, and Intelligence," Child Development, 27:279-382, 1956.

anxiety and arithmetic were highest in the top grades of the school. For boys, the reported correlation was -.74 while for girls the reported figure was -.57, both of which were highly significant at the .01 level.

Other studies which report relationships between attainment and anxiety also take into account intelligence. Lynn and Gordon summarized the relationship of neuroticism and extroversion to intelligence and educational attainment. They report no significant correlation between intelligence as measured by the matrices test and anxiety. Further investigations in the United States frequently report negative correlations between anxiety and attainment while English studies tend to suggest that the relationship is positive. The reports of the relation between anxiety and attainment is evidently in confusion. They summarize their conclusions as follows:

> In view of the many conflicting reports on the relation of drive to performance, the most important finding of the present investigation is probably that of the curvilinear relations between neuroticism and scores on the matrices. It confirms the theory that for this type of task, there is a golden mean of neuroticism or anxiety, neither too little nor too much being desirable for efficient performance.[31]

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[31]R. Lynn and T. E. Gordon, "The Relation of Neuroticism and Extroversion to Intelligence and Educational Attainment," British Journal of Educational Psychology, 3:201, June, 1961.

The moderately high correlations which have been consistently reported between various measures of intelligence and college grades would seem to indicate that poor academic performance was primarily determined by limited ability while good grades were largely determined by superior intellectual endowment. Thus personality or motivational variables might be most likely to influence the academic performance of students with average ability. The purpose of a study conducted by Spielberger and Katzenmeyer was to examine additional evidence of the relationship between anxiety scores and grade-point averages and to determine whether the relationship varied as a function of the intellectual level of the student.[32] The subjects involved were 640 men enrolled in the introductory course in psychology at Duke University.

When intelligence was not taken into account, there was a small inverse relationship between anxiety scores and grade-point averages. In order to determine whether this relationship was influenced by the intellectual ability of the subjects, the total sample was divided into three groups (lower 20 per cent, middle 60 per cent, upper 20 per cent) on the basis of ACE scores. Tests for linearity and curvilinear-

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[32] C. D. Spielberger and W. Katzenmeyer, "Manifest Anxiety, Intelligence, and College Grades," Journal of Consulting Psychology, 23:275-278, 1959.

ity of regression indicated that grades were unrelated to anxiety scores for the high and low intelligence groups. However, for the middle group a significant inverse relationship existed between grade-point averages and anxiety level. These results seem to indicate that college work was too difficult for low aptitude students while high aptitude students tend to get good marks regardless of their anxiety level.

Stakenas examined the long term effect and the short term effect of anxiety arousal in academic achievement situations.[33] The sample consisted of university students enrolled in an introductory course in economics or psychology. He concluded that no general statements should be made regarding the effect of anxiety on performance without also specifying characteristics of the tasks as well as relevant characteristics of the subjects. Whether or not anxiety is disruptive depends upon the amount of learning that must be acquired in order to perform the task and this amount of learning depends upon the subject's previous familiarity with the task.

Reese administered the children's form of the Manifest Anxiety Scale, an arithmetic test, and an intelligence

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[33] R. G. Stakenas, "Evaluative Stress, Fear of Failure, and Academic Achievement," Dissertation Abstract, 27:601-A, No. 2.

test to 539 students in the fourth- and sixth-grades.[34] He found that manifest anxiety was inversely and monotonically related to the number of correct responses on the achievement tests. Partialing out I.Q. had little effect on the correlations between manifest anxiety and achievement, but predictions of achievement were not appreciably improved by the combination of manifest anxiety and intelligence.

There is another emotional factor in mathematics which appears to possess some primary as well as secondary characteristics in that it is pathological in intensity and at the same time specific to mathematics. It seems to go very much deeper into the personality pattern than do simple feelings of distaste, and it also appears to be much more closely related to basic personality variables, anxiety in particular. This is what Dreger and Aiken call number anxiety.[35]

Now it is evident that this is not the same factor as causes the poor mathematical performance of the anxious or unstable person, not that which intervenes in the person who has poor attitudes to the subject. The number anxious child, or adult, is by definition, deeply disturbed when he is

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[34] H. W. Reese, "Manifest Anxiety, Achievement, and Test Performance," Journal of Educational Psychology, 52:132-135, 1961.

[35] R. M. Dreger and L. Aiken, "The Identification of Number Anxiety in a College Population," Journal of Educational Psychology, 48:344-351, 1957.

dealing with number and arithmetical operations: the child who only dislikes arithmetic, however thoroughly, may not feel particularly anxious or insecure in his performance.

Dreger and Aiken hypothesised that number anxiety was a factor distinct from general anxiety, that it is uncorrelated with I.Q., and that the number anxious person would perform at a lower level of mathematical performance than non-number anxious persons of the same intelligence. The Taylor anxiety scale, with three additional number items, was administered to a student population and from the results, subgroups of students were selected at random from each of the four extremes of the scores. Each subgroup was tested with the Wechsler intelligence scale and with each subtest, a galvanic skin response reading was taken. Only on the arithmetic subtest were there any significant readings on the skin test, and these clearly differentiated the number anxious from the non-number anxious subjects. Intercorrelations between number test items were high: those between number and general anxiety were low: thus suggesting that number anxiety is a separate factor in its own right. There was no clear relationship between number anxiety and intelligence, but there was a significant negative correlation with results on university mathematics examinations. The authors, on the first point, conclude that low intelligence per se is not a factor

causing number anxiety. This important study, however, can only be regarded as suggestive especially in view of the very small number of test items and because the subjects were undergraduates, not children.

A summary of the research done on anxiety points out the following conclusions:

1. There seems to be no clear relationship between anxiety scores and intelligence scores.
2. Anxiety markedly affects attention and concentration but there is insufficient evidence to say what happens when conceptual thinking is involved.
3. The effects of anxiety on academic attainment are not consistent as pointed out by studies in England and the United States. The apparent inconsistencies in all these results may be explained to a certain extent by the curvilinear relationship between anxiety and performance. When anxiety is low, performance tends to be poor simply because there is a level when the subject "couldn't care less" and obviously in this condition he will not perform at his optimum level of efficiency. On the other hand, under extreme conditions of anxiety, performance often becomes utterly disorganized.
4. When a task or stimulus is clearly defined, highly-anxious people may function excellently, but in a vaguely

defined situation, anxious people manifest a poorly organized performance.

5. Many studies report a relationship between maladjustment and poor mathematical performance. Although this is also true of some other school subjects, this relationship seems to be much closer in the case of arithmetic.

Attitude

Among the secondary emotional factors related to mathematical performance are the negative and positive attitudes specifically directed towards mathematics. Where do these attitudes have their origins? What causes them? What influence do teachers and parents have on students' attitudes toward mathematics? These questions deserve consideration if we are to assess the effect of attitudes upon mathematical performance.

Dutton claims that most of the children he tested felt they knew when their feelings towards arithmetic were developed.[36] Grades three to eight and especially grades five and seven were most significant for attitude development. However, it must be remembered that this information was obtained on the basis of the pupil's own memory--which is not

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[36] J. H. Dutton, "Attitudes of Junior High School Pupils," School Review, 64:18-22, January, 1956.

likely to be entirely reliable, especially during the earlier school years. Dutton also asked them if their attitudes had changed at all in the course of their own school years, and about one-third of the children claimed that they had probably done so. This data collectively would suggest that attitudes as measured by Dutton are very much a function of the immediate syllabus and may not be particularly deep-rooted.

Churchill, reported in Biggs, seems to be closer to the mark by suggesting that faulty development of number concepts is probably the most likely cause of the strong dislike and even fear which many adults show towards arithmetical operation.[37] This dislike or fear has its foundations in elementary school where children are taught to calculate without the understanding of the significance of number relations.

Related to the question of the origin of attitudes to arithmetic is the more important one of the causes of these attitudes. Some of these are self-evident. The "don't like" attitude of some students towards mathematics was often the result of trying too much too quickly, with resultant disillusionment and failure. Mace's opinion is that this same

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[37]J. B. Biggs, "Attitudes to Arithmetic--Number Anxiety," Educational Research I, 3:9, June, 1959.

phenomenon is due to too much abstraction too early.[38] Drummond writes that sensitive, intelligent children, if they are hurried or puzzled or allowed to feel that they have made fools of themselves, will give up numbers and seek success in some other sphere.[39]

Valuable as these opinions are, however, they are opinions only. What do students give as reasons for liking or disliking mathematics? According to Dutton, when junior high school pupils were asked their reasons for disliking arithmetic they listed lack of understanding, too difficult and complicated, poor achievement, and boring and repetive.[40] Students listed practicality, interest, and challenge of arithmetic as the reasons for liking it.

Pritchard shows that boys and especially girls, dislike arithmetic because of a feeling of incapacity and strain when dealing with difficult items in the curriculum.[41] Free-

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[38] C. A. Mace, The Psychology of Study (London: Meuthen and Co., Ltd., 1932)

[39] M. Drummond, Psychology and Teaching of Number (New York: World Book Company, 1922).

[40] Dutton, op. cit., pp. 18-22.

[41] R. Pritchard, "The Relative Popularity of Secondary School Subjects at Various Ages," British Journal of Educational Psychology, 5:157-179, 1935.

man bears this out almost exactly--inability to master technical difficulties he quotes as the most frequently expressed reason for not liking arithmetic.[42] Freeman does note, however, a general improvement in attitudes over the years, and it has been suggested that this may be due to a lightening of the syllabus and improvement in teaching techniques apparent since Pritchard's work in the 1930's.

Poffenberger and Norton place a lot of emphasis on the influence of parents which they claim determines the child's initial attitude and affects his achievement.[43] If parents themselves had trouble with mathematics, then so does the child, although this may not apply where they have favourable attitudes to their child's study generally. These same authors also claim that the personality of the teacher and whether he is liked or disliked will influence children's attitudes.

Aiken and Dreger correlated the attitude scores of approximately three hundred college students with the re-

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[42] W. A. Freeman, "An Enquiry into the Attitudes of Secondary School Boys and Girls Towards Arithmetic," (unpublished Master's Thesis, University of Birmingham, Birmingham, 1948.)

[43] T. Poffenberger and D. Norton, "Factors Determining Attitudes Towards Mathematics and Arithmetic," The Arithmetic Teacher, 3:113-116, 1956.

membered characteristics of their former teachers.[44] A positive relation was shown to exist. However, they found no evidence that mathematics attitudes of students are related to the students' perception of the parents' attitude toward the subject. These results seem to contradict the findings of Poffenberger and Norton in 1959 when they found that only the parents' attitude as perceived by the students were significantly related.[45] Students with favorable attitudes towards mathematics reported their parents liked mathematics and expected them to achieve well in it. The attitude towards the teacher or influence of the teacher was only slightly related with student attitude scores and many students reported liking the teacher but not the subject.

Most people would imagine that there is a close relationship between dislike of teacher and dislike of subject. Indications from research show that this relationship is not clearly established. Pritchard asked a section of this sample if their best-liked subjects were taught by their best-liked teachers, and similarly for their worst-liked subjects.

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[44] Lewis R. Aiken and Ralph M. Dreger, "The Effects of Attitude on Performance in Mathematics," Journal of Educational Psychology, 52:19-24, February, 1961.

[45] T. Poffenberger and D. Norton, "Factors in the Formation of Attitudes Towards Mathematics," The Journal of Educational Research, 52:171-177, 1959.

The weight of the evidence was emphatically against the opinion that the popularity or unpopularity of the teacher is the main influence.[46] Other researchers, including Lovell and White agree that the attitude the pupil exhibits towards his teacher does not appear to have more than small influence upon the pupil's attitude towards arithmetic.[47]

Biggs reports finding from a pilot study that amongst a group of fifty-four third-year junior school children, there was a significant correlation between liking for teacher and attitudes to arithmetic in the case of boys in the sample.[48] When it came to arithmetic performance, there was no significant correlation between boys' attitude to teacher and test results. For girls, non-significant correlations were obtained in both attitude to teacher with attitude to arithmetic, and attitude to teacher and arithmetical performance.

Let us now consider the effects that student attitudes have upon mathematical performance. Hungerman, in a study cited earlier, investigated the relationship between con-

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[46] Pritchard, loc. cit.

[47] K. Lovell and G. White, "Some Influences Affecting Choice of Subjects in School and Training College," British Journal of Educational Psychology, 28:15-24, 1958.

[48] J. B. Biggs, "Attitudes to Arithmetic--Number Anxiety," Educational Research I, 3:11, June, 1959.

ventional arithmetic achievement, contemporary mathematics achievement, and attitudes towards mathematics.[49] She found attitude towards mathematics was positive for both treatment groups and appeared to be less a function of achievement or the type of mathematics program studied than might have been expected. Correlations of the order .2 and .3 were reported between attitude and achievement.

Johnson reported, that next to intelligence, student attitude proved the most closely related to achievement.[50] She also found student attitude was of significance in all three American groups but only in the British middle group.

Remai investigated the attitudes towards mathematics of a group of modern mathematics students and a comparable group of traditional mathematics students at the grade nine level.[51] He found no significant differences in the attitudes displayed by students in these two groups. A positive relationship existed between attitude towards mathematics and problem solving skill for each of the two programs. A correlation of.27 was given for the modern group as compared to a correlation of .26 for the traditional group.

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[49] Hungerman, loc. cit.

[50] Johnson, loc. cit.

[51] H. A. Remai, "An Experimental Investigation Comparing Attitudes Towards Mathematics of Modern and Traditional Mathematics Students at the Junior High School Level," (unpublished Master's Thesis, University of Alberta, Edmonton, 1965).

A Thurstone-type attitude scale, devised by Ellingson, was administered to 755 high school students representing various types of mathematics classes.[52] The correlation reported between teacher grades and the attitude scores was .39.

In a small study conducted in the National Foundation for Educational Research program, a third-year junior school sample was administered a sentence completion test and the attitudes to arithmetic and English thus obtained were correlated with attainment scores in these subjects.[53] The correlation between attitude and arithmetical performance was .41 while a correlation of .15 was reported between attitude and achievement in English. Only the former correlation is significantly different from zero. Because the sample was small, these results must be considered as suggestive only. Yet the findings do support the work of other researchers which shows that the pupil's attitude to his work in arithmetic has more bearing upon his actual attainment than is the case in other subjects.

From the foregoing discussion, we may conclude that attitudes towards mathematics appear to arise very early

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[52] J. B. Ellingson, "Evaluation of Attitudes of High School Students Towards Mathematics" (unpublished Doctoral Dissertation, University of Oregon, Eugene, 1962).

[53] Biggs, op. cit., p. 14

in a child's school career, but they may be deepened or changed probably at any point during the child's school career. The most common reason for negative attitudes are based upon a lack of proper understanding of the subject and plain boredom. This lack of understanding and apparent boredom may well be a product of the teaching method as well as the material itself. The extent of parental and teacher influence upon student attitude development is not conclusively known. Not unexpectedly, children with poor attitudes to arithmetic do not perform as well as those who like the subject. Moreover, the influence of attitudes upon performance is greater in arithmetic than in English or other subjects. The child who dislikes English may yet achieve high marks in that subject--this is not so likely to occur in arithmetic.

A review of the literature relevant to this study has been presented in this chapter. Among the topics reviewed were proposed mathematics curricula for high school students, opinions and evaluations of various mathematics curricula, and factors related to the learning of mathematics.

CHAPTER III

THEORY AND DESIGN OF THE STUDY

I. OVERALL PLAN OF THE STUDY

The major purpose of this study was to evaluate three mathematics half-courses by analyzing data assembled from different sources. The data was collected by using tests and questionnaires. Measures of student ability and achievement were obtained by administering appropriate tests to the students while measures of student attitude and anxiety were determined by using a questionnaire. A second questionnaire solicited opinions regarding the half-courses from professors at the University of Alberta while a third was prepared and sent to the teachers who taught the half-courses.

Since much of the data used in this study was obtained through questionnaires, the theoretical considerations used in the construction of these instruments are reported. In effect, the views of various writers were incorporated into a theoretical framework upon which each questionnaire was based. This chapter describes the development of these questionnaires as well as the nature of the samples, collection of data, statement of hypothesis, and statistical procedures relevant to this study.

II. THEORETICAL JUSTIFICATIONS FOR QUESTIONNAIRES

Attitude-Anxiety Questionnaire

To determine the attitude of the students toward mathematics and the anxiety produced by these courses, an attitude-anxiety questionnaire was constructed. This instrument was designed so that some of the items pertained to attitude measurement while others pertained to anxiety measurement. The theory behind the construction of each type of item is now given.

Attitude items. There is little doubt that the determination of students' attitudes toward mathematics is an important part of curriculum evaluation. While analyzing new mathematics programs, one educational group stated:

> For our purpose, we must assume that program changes are aimed at improving the achievement of the following objectives: developing greater computational skill, developing the ability to think mathematically, developing favourable insight into the structure of mathematics, developing favourable attitudes toward mathematics.[1]

Another writer also expressed his concern about the amount of attention given to mathematical attitudes when he wrote:

> In our concern for improving the mathematics curriculum and increasing enrollment in mathematics, have we forgotten a crucial factor, namely attitudes? Have we

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[1]National Council of Teachers of Mathematics, An Analysis of New Mathematics Programs, 1964, p.37.

> forgotten that learning involves emotional vectors such as attitudes? It is the attitudes that our students develop which are likely to stimulate or to stop further study of mathematics. It is the attitudes which we build that are highly involved in the learning and retention of our subject.[2]

The theory base for constructing attitude items was derived from a study made by Remai who developed an attitude scale based upon an extensive framework supplied by Krathwohl, Bloom, and Masiu.[3,4] The major features of this framework are as follows:

> Many educational objectives are stated in such a manner that they are meaningless to the educators who are trying to achieve them. To remedy this situation Krathwohl developed a large over-all scheme into which the educational objectives could be placed. The purpose of placing the objectives within the classification scheme was to position them on a continuum and thus serve to indicate what was intended by each objective. This would also help to clarify language of educational objectives by stabilizing the meaning of various terms.
>
> Internalization was selected as the organizing principle for the taxonomy since it helps delimit, describe, and classify objectives into the desired structure. It describes a process whereby a given phenomenon passes from a level of bare awareness to a position where it guides and controls the behavior of a person. Internalization is analagous to an old education axiom that states "growth occurs from within".

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[2]Donovan A. Johnson, "Attitudes in Mathematics Classrooms," <u>School Science and Mathematics</u>, 57:113, 1957.

[3]H. A. Remai, "An Experimental Investigation Comparing Attitudes Toward Mathematics of Modern and Traditional Mathematics Students at the Junior High School Level" (unpublished Masters Thesis, University of Alberta, Edmonton, 1965).

[4]David R. Krathwohl, Benjamin S. Bloom, and Bertram B.

Internalization refers to this inner growth which takes place as the individual accepts attitudes, codes, principles, or sanctions that become a part of him in forming value judgments or in determining his conduct. This growth takes place in different ways. One of these ways is the increased emotional impact of the experience. At the lowest levels of the internalization continuum there is little emotion in the behavior since the individual is merely perceiving the object. At the middle levels, emotional response is a recognized and critical part of the behavior as the individual actively responds for the emotion that is experienced determines the type of overt behavior. This emotion decreases as the behavior becomes completely internalized and routine.

This growth along the continuum may also be traced by the relationship between external and inner control. At the lowest levels of the continuum inner control merely directs attention. At middle levels, inner control produces the proper responses at the bidding of an external force. At the highest levels, inner control produces appropriate responses in the absence of an external force.

The titles of the categories imply this change in direction of control and emotion. A lower level is titled "Responding", thus indicating that the individual is reacting to an inner control with some emotion. The next level, "Valuing" indicates that the control is becoming more internalized with more emotion involved. However, in the next two categories, the emotional component decreases but the inner growth continues to become more completely internalized.[5]

Anxiety items. Developing some theoretical justifications for the construction of items designed to measure students' anxiety associated with the half-courses posed a

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Masiu, Taxonomy of Educational Objectives, The Classification of Educational Goals, Handbook II: Affective Domain (New York: David McKay Company, Inc., 1956).

[5]Remai, op. cit., pp. 11-13.

more difficult task. The main reason for this difficulty stemmed from the nature of anxiety itself and the type of research conducted involving anxiety.

Berg claims that anxiety is a state of tension that arises in an organism in consequence of frustration of instinct gratification and subsequently in anticipation of this.[6] Mowrer says that anxiety is the apprehension of a threat to some value that is vital to the individual's security.[7] Anxiety has also been considered to be the product of motivation and frustration. Other terms such as drive, stress, neuroticism, frustration, fear, and motive have been equated with anxiety. Biggs sums up these views as follows:

> ....in the group of studies concerning the relationship between anxiety and performance, some are concerned with the effects of specific stresses in normal or presumably normal individuals; others compare the performance of chronically anxious individuals with that of non-anxious, using clincal ratings, projective tests, pencil and paper questionnaires or some other psychometric technique as the criterion; and yet others compare psychometrically defined "anxious" with "non-anxious" subjects in both stressed and unstressed conditions. It is evident from the above discussion that the term "anxiety" may have a different meaning, depending on which of the three experimental situations is being considered at the time.[8]

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[6] C. Berg, "Amended Definition of Anxiety," British Journal of Medical Psychology, 25:158, 1952.

[7] O. H. Mowrer, Learning Theory and Personality Dynamics (New York; Ronald Press, 1950).

[8] J. B. Biggs, Anxiety, Motivation and Primary School Mathematics Occasional Publication No. 7 (The Information Service of the National Foundation for Educational Research in England and Wales, 1962), p. 54.

However, one of the purposes of this study was to determine the level of anxiety associated with each half-course. This relationship deserves consideration since many of the changes in the mathematics curriculum involves exposing students to more advanced concepts at an earlier age. This leads to what Biggs has to say:

> There is, it seems, usually a very good reason why an individual becomes anxious in the sort of situation in which he is required to do something at a certain level and he is not sure whether or not he will be able to do it--he questions, that is, his "cognitive integrity" and sees the situation as being threatening to his own self-esteem. If he does not seriously doubt his ability to do that task--or on the other hand is equally certain that he will not be able to do it--the situation is not anxiety arousing and on any valid questionnaire assessing this kind of situational anxiety, this individual would obtain a low score, irrespective of his anxiety in other situations.[9]

This quote explains the type of anxiety that the investigator attempted to measure in this experiment. The amount of anxiety determined was that exhibited by the students participating in the half-courses under normal classroom situations. The basic assumption underlying this measurement is that any differences in mean anxiety shown by the students in the various groups is related to the half-course being studied.

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[9] Ibid., p. 120.

A number of guidelines were followed as the anxiety items were constructed. A consideration of Freud's three criteria of the anxious reaction served as one guideline. These criteria are: (1) it is unpleasant, (2) there are physiological concomitants, and (3) it involves conscious awareness.[10]

Lighthall's characteristics of the anxiety effect were also used.[11] He claims anxiety is not merely an heightened emotion but it is itself unpleasant, and when intense is painful and fatiguing. Also it does not occur in any single intensity but ranges on a continuum from very weak to overwhelming terror.

A set of directly observable anxiety symptoms as given by De Michele was also employed.[12] The symptoms include great tension, great difficulty in concentration, great difficulty in speaking or conversing, poor sleep habits, great fear, and others.

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[10] S. Freud, Inhibitions, Symptoms, and Anxiety (London: Hogarth Press, 1948).

[11] Frederick E. Lighthall, "Anxiety as Related to Thinking and Forgetting," What Research Says to the Teacher, No. 30 (National Education Association of the United States, May, 1964).

[12] John H. De Michele, "The Interpretation of Anxiety By Various Psychotherapeutic Schools," Journal of Consulting Psychology, 18:47-52, 1954.

University Questionnaire

The second instrument developed for use in this study was a questionnaire which was sent to a sample of university professors representing various departments and faculties at the University of Alberta. The main purpose of this questionnaire was to determine, on the basis of content only, which of the three half-courses was the most beneficial for freshman university students.

There were two major reasons why the university questionnaire was prepared. Firstly, Mathematics 31 is a compulsory course for any student entering the University of Alberta with a major in engineering or the honors programs in the physical sciences. It is strongly recommended for any student entering the general physical sciences programs. One obvious objective of the Mathematics 31 program, therefore, is to prepare the student for university studies. Fehr sums this up very well when he says:

> Mathematics study in the secondary school has preparatory value, not only for the collegiate study of pure mathematics, but also for engineering and applied sciences. The content of mathematics should then be judged also in light of its subsequent use in college courses other than mathematics alone.[13]

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[13]Howard F. Fehr, "Teaching High School Mathematics," What Research Says to the Teacher, No. 9 (National Education Association of the United States, October, 1955), p. 12.

Secondly, university personnel and particularly mathematicians, are actively engaging in mathematics curriculum reform at the secondary school level. This is evidenced by committees such as the School Mathematics Study Group, the University of Maryland Project, and the University of Illinois Committee on School Mathematics. Goodlad claims that the present curriculum is mainly discipline-orientated and that the ends and means of schooling are now to be planned by physicists, mathematicians, and historians to assure authenticity of content.[14]

Teacher Questionnaire

The third questionnaire used in this study was circulated to teachers involved in the Mathematics 31X program to solicit their evaluations of the experimental half-courses. Although the questionnaire was not designed to be analyzed using rigorous statistical methods, the participating teachers were in an excellent position to provide useful information regarding these half-courses. This is in keeping with Cronbach's definition of evaluation as "the collection and use of information to make decisions about an educational pro-

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[14]John I. Goodlad, "Direction and Redirection for Curriculum Change," Curriculum Change: Direction and Process, R. R. Leeper (ed.), (Washington, D.C.: Association for Supervision and Curriculum Development, 1966).

gram."[15] Cronbach goes on to say:

> Evaluation is too often visualized as the administration of a formal test, an hour or so in duration, at the close of a course. But there are many other methods for examining pupil performance, and pupil attainment is not the only basis for approving a course.[16]

To assist the investigator in preparing this questionanire, Johnson's criteria for evaluating a school mathematics curriculum was consulted.[17] Johnson claims that the five major criteria to be followed are: mathematical criteria, psychological criteria, pedagogical criteria, philosophical criteria, and measurement criteria.

A mathematics course meets the mathematical criteria if its content is good mathematics. That is, the content is presented sequentially in a correct and precise manner using clear terminology and symbolism. The mathematical content should also be appropriate in that it meets the needs of the students by emphasizing flexibility, procedures, broad principles, and structures rather than number facts and skills.

Psychological criteria refers to the conditions surrounding the learner. Is the student ready for the concepts that are to be introduced? Do these concepts have meaning for the learner? Does the child show a willingness to learn?

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[16] Ibid., p. 239

[17] Donovan A. Johnson, "Evaluating a School Mathematics Curriculum," School and Society, 90:424-426, December, 1962.

Pedagogical criteria refers to the extent to which the course can be taught in the present school situation. Teachers with an adequate background must be available and sufficient time must be provided for adequate presentation of the topics outlined. Adequate physical facilities and materials such as textbooks, teacher guidebooks, and other instructional aids must be provided.

Another crucial basis for evaluation involves philosophical questions. Curriculum planners must consider the immediate purposes of mathematics instruction as well as where they are going and what they are striving to obtain. The age-old question of what they should teach and whether it is more important than something else also comes under this category.

Johnson's final criteria involves the need for improving measuring and testing devices that can be used to render judgment on the relative effectiveness of a given curriculum. There is also a need to develop instruments to test some of the stated objectives. Long-range evaluation in terms of continued study in mathematics, success in related scientific fields, success in selected vocations, and success in citizenship must be undertaken.

III. CONSTRUCTION OF THE QUESTIONNAIRES

On the basis of the theoretical considerations outlined previously, the three questionnaires were constructed. The outline of the procedures that were followed are given in this section.

Attitude-Anxiety Questionnaire

Attitude items. The attitude items developed by Remai were used in this study because they were developed from a suitable theoretical framework and measures of reliability and validity were reported.[18] Modifications were necessary since Remai's scale was intended for grade nine students while the present study attempted to measure attitudes toward a specific branch of grade twelve mathematics. A copy of Remai's scale can be found in Appendix A.

Remai's scale is an example of a Likert-type scale. A Likert-type scale is constructed very similar to a multiple-choice test in which the student reads the stem of the question and then selects the response which best suits him.[19]

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[18] Remai, op. cit., pp. 52-56.

[19] Claire Sellitz, Marie Jahoda, Morton Deutoch, and Stuart W. Cook, Research Methods in Social Relations, (United States: Holt, Rinehart and Winston, Inc., 1963). pp. 366-369.

The responses, usually three or five in number, are weighted according to direction and intensity as judged by the writer. For Remai's scale, in which five responses were used, the most favourable response was weighted five and the least favourable response was weighted one. The student's attitude score is obtained by summing the weights for each individual item.

Anxiety items. A Likert-type scale consisting of twelve anxiety items was constructed. Three of these items were similar to the ones used by Dreger and Aiken in their study.[20] Other items similar to some found in Sarason's test anxiety scale for children were also included on the scale.[21]

In addition to the procedures outlined above, all the anxiety and attitude items were submitted to mathematical educators and graduate students for criticism. A list of the eighteen attitude items and twelve anxiety items used in the pilot study are given in Appendix B.

Pilot study. The initial form of the attitude-anxiety questionnaire was administered to sixty grade twelve students

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[20] Ralph M. Dreger and L. R. Aiken, "The Indification of Number Anxiety in a College Population," Journal of Educational Psychology, 48:344-351, 1957.

[21] S. R. Sarason, K. Davidson, F. Lighthall, and R.

enrolled in the usual Mathematics 31 course on February 21, 1967. The questionnaire was designed so that it could be machine scored and the students were requested to put their names on the answer sheets. The students were assured that their responses would be kept confidential and the teacher was not present when the questionnaire was given.

The answer sheets were machine scored and the attitude and anxiety items were analyzed separately. Statistical procedures were than applied to determine measures of reliability and validity for the items. The method of item-total score correlation was used. That is, the score for each item was correlated with the total score using the Pearson product-moment formula. As Guilford points out, the larger the correlation the higher the construct validity of the item. He says:

> More important than the matter of item difficulty are the questions of whether a test item discriminates individuals in line with other items in the test, whether responses to the item predict some criterion, and whether the criterion is total score on the test of which it is a part or some outside evaluation in individuals. The problem is often known as that of item validity; a case of construct validity when the criterion is the total score, and a case of predictive validity when the cri-

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Waite, "A Test Anxiety Scale for Children," Child Development, 29:105-113, 1958.

terion is an outside measure, particularly when it is a practical variable of some kind. Of course, raising the correlations of items with total score has the effect of increasing homogeneity of the test, which is a matter of reliability, But, just as stated, it is also a matter of construct validity, i.e., ensuring that the psychological variable measured be uniform for all items.[22]

Guilford goes on to say:

> To be more specific, the item-test correlations for well-constructed items range between .30 and .80 which means item intercorrelations approximately between .10 and .60. Items within these ranges of correlation should provide tests of both satisfactory reliability and validity. There is probably better reason for going below these limits than above them in constructing items. To do so would probably error on the side of validity, which, after all, is the most important.[23]

Appendix C contains correlation matrices showing the item-test correlations as well as the intercorrelations for the attitude and anxiety items. Inspection of these matrices show that most of the values fall within the suggested boundaries.

One measure of external criteria was also used. All of the students participating in the pilot study were rated by the same teacher on the attitudes that they showed towards Mathematics 31. To assist the teacher in rating the

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[22]J. P. Guilford, <u>Fundamental Statistics in Psychology and Education</u> (New York: McGraw-Hill Company Inc., 1965), pp. 497-498.

[23]<u>Ibid</u>., p. 481

students, Remai's guideline was used.[24] A copy of this guideline is found in Appendix D. The correlation between the teachers rating and the students attitude scores produced a predictive validity coefficient of 0.57 which is significantly different from zero at the .01 level.

The teacher was also asked to rate each student according to the amount of anxiety the student exhibited during Mathematics 31 classes. A guideline, which was prepared to help the teacher with this task, is contained in Appendix E. The correlation between the teacher's ratings and the students anxiety scores produced a predictive validity coefficient of -0.16. This insignificant correlation is not surprising since the teacher did express that he had encountered some difficulty in rating the students on this variable.

Two months after the first administration of the attitude and anxiety questionnaires, they were re-administered to the same students. A test-retest reliability coefficient of 0.82 was obtained for the attitude items and a coefficient of 0.88 was obtained for the anxiety items. The size of the test-retest coefficient indicates the stability of the

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[24]Remai, op. cit., p. 93.

trait being measured as well as the precision of the instrument.[25]

Correlations were computed between the attitude scores and the anxiety scores for the administration of the questionnaires. Values of -0.54 and -0.63 were obtained. These results are similar to those reported by Phillips who found that negative attitudes towards school were significantly related to anxiety only amongst highly intelligent children and who were presumably good achievers.[26]

On the basis of the statistical procedures outlined above, three attitude items were deleted from the scale. Otherwise, the remaining fifteen attitude items and the twelve anxiety items were incorporated into the final attitude-anxiety questionnaire which is given in Appendix F.

University Questionnaire

The university questionnaire contained four questions, the first of which asked the professors to indicate their area of instruction. The second question requested these educators to indicate which of the half-courses they thought

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[25]Robert L. Ebel, Measuring Educational Achievement (Englewood Cliffs, N.J.: Prentics-Hall Inc., 1965) p. 467.

[26]B. N. Phillips, E. Hindsman, and E. Jennings, "Influence of Intelligence or Anxiety and Perception of Self and Others," Child Development, 31:41-46, 1960.

was the most beneficial for freshman university students intending to major in the area of instruction indicated in question one. The professors were asked to rank the courses (one, two, or three with one indicating the most preferred) or to state no preference. For the third question, the professors were asked if they considered it essential that freshman students have a knowledge of the course that they ranked number one before the students entered university. An answer of yes or no was requested. A fourth, open-ended question was provided to allow the professors to comment on the half-courses.

Accompanying the questionnaire was a cover letter and a copy of each of the half-course outlines. The cover letter explained the project and the outlines provided a detailed account of the prescribed content for the half-courses. The questionnaire was purposely kept brief and to the point to encourage responses. A guide for constructing the questionnaire was supplied by Sellitz.[27] Appendix G contains a copy of this questionnaire and the accompanying material.

Teacher Questionnaire

A rough draft of the teacher questionnaire was sub-

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[27] Sellitz *et al.*, op. cit., pp. 546-574.

mitted to the Senior High School Mathematics Subcommittee on May 4, 1967. Four members of the committee were each given a copy of the questionnaire and were asked to make any suggestions that they considered would improve the instrument. The returned questionnaires indicated the need for greater clarity and precision so that many changes were then made with questions being added, deleted, or modified. A copy of the final draft of the teacher questionnaire is located in Appendix H.

IV. NATURE OF THE SAMPLES

Student Sample

The total number of students involved in this study was 325, of which 232 were in the experimental groups and 93 in the control group. Ideally, it would have been desirable to have a random sample of students and teachers for each treatment group, but since the study was conducted in a practical school situation and since one determining factor was the availability of teachers willing to participate in the experiment, such an assignment was not possible.

Further limitations were also placed on the sample. For various reasons, two of the fourteen experimental classes were not allowed to answer the attitude-anxiety questionnaire. Although some difficulty was experienced in finding sufficient

students to participate in the control group two classes were located. To render a control group of appreciable size those Mathematics 31 students who had participated in the pilot study along with the aforementioned classes constituted the control group. Table I shows the composition of the student sample used in this study.

TABLE I

COMPOSITION OF STUDENT SAMPLE PARTICIPATING IN STUDY

Group	Number of Classes	Number of Students
Calculus	5	85
Matrices	4	65
Probability	3	82
Control	5	93
Totals	17	325

University Professor Sample

The university questionnaire was mailed to ninety-eight professors instructing in seven major areas at the University of Alberta. These areas were: Engineering, Physical Sciences, Biological Sciences, Mathematics, Humanities, Social Sciences, and Education. Some of these areas included a number of different departments. For example, the Social Sciences included anthropology, history, philosophy, economics,

political science, geography, and sociology. By sampling these major areas a good cross-section of opinion on the importance of the three experimental half-courses for university study was obtained. Fourteen questionnaires were mailed to professors instructing in each of the seven major areas.

A sampling procedure known as stratified random sampling was used to select the desired number of professors. Under this procedure, the population is first divided into a number of classes or strata. A random sample is then taken from each stratum and the subsamples are then joined to form the total sample. In choosing a random sample from each stratum, a table of random numbers as outlined in Kenney was employed.[28]

The seven areas which provided the main strata were subdivided further to ensure that all departments were sampled as equally as possible. A list of the academic staff at the University of Alberta was obtained by using the Directory of Academic and Administrative Staff, University of Alberta, 1966-67.

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[28]J. F. Kenney and E. S. Keeping, Mathematics of Statistics (Princeton, N.J.: D. Van Nostrand Company, Inc., 1954), pp. 205-207.

Teacher Sample

The methods used to select the teachers for the Mathematics 31X program varied. Some of the teachers were approached directly by the Subcommittee, others were approached by their local superintendent or inspector of high schools, while others applied to teach the experimental courses. However, no teacher was forced to take part in the program. The participating teachers were then assigned to teach one of the experimental half-courses. As a result, the students involved in the program had registered for Mathematics 31 at the start of the school year and were then informed by the teachers that they were part of the Mathematics 31X program. In some cases, especially where the high school had more than one Mathematics 31 class, the students had the opportunity to decide whether or not they wished to participate in the experimental program. If the students decided to participate in the experimental program, they had no choice in determining which of the new half-courses they would study.

All fourteen teachers that took part in the Mathematics 31X program completed the teacher questionnaire.

V. COLLECTION OF DATA

The collection of data pertinent to this study extended over a period of approximately six months. In Jan-

uary, 1967, the Cooperative School and College Ability Test (SCAT), Level 1B, was administered to all grade twelve students in the Province of Alberta. The SCAT scores for those students engaged in this study were then obtained from the Department of Education.

Buros reports the SCAT test to have a Kuder-Richardson (Formula 20) estimated total reliability score of 0.95.[29] He also describes the test, which contains a verbal ability score and a quantitative ability score, as being non-specific and indicative of a student's ability at school work rather than his actual achievement level. Throughout this study only the total raw scores were considered.

As mentioned previously, all students involved in the Mathematics 31X program studied trigonometry for the first five months of the school year. Late in January, 1967, these students wrote a final test on this material. The total possible raw score for this test was eighty-seven. These raw scores were also obtained from the Department of Education.

The attitude-anxiety questionnaire was administered to all participating students (except some of those already

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[29]Oscar K. Buros (ed.), The Fifth Mental Measurements Yearbook (Highland Park, N.J.: The Gryphon Press, 1959), pp. 453-455.

mentioned in the control group) during the month of May, 1967. In all cases, the investigator administered the questionnaire and the classroom teacher was requested to leave the room while the questionnaire was being completed. The questionnaire was administered under normal classroom conditions during the regular Mathematics 31X period. Before each administration, the purpose of the questionnaire and how it was to be completed was explained to the students. The students were requested to refrain from discussing any of the items until everyone was finished. As in the pilot study, the students were assured that their individual results would be kept confidential.

The questionnaires administered to the control and experimental groups were identical except for the wording of the question stems. For the control group, the questions were worded to refer to the Mathematics 31 course, while for the experimental groups the questions were worded to refer to the Mathematics 31X course. Students in the experimental groups were then instructed to answer the questions on the basis of the experimental half-course they were currently studying.

At the completion of the half-courses each group wrote a final test on the material that it had studied since February. The calculus group wrote a common test, as did

the probability and matrices groups. These tests were constructed and marked by the participating teachers and the raw scores were forwarded to the writer.

On March 14, 1967, the university questionnaire was mailed to those professors included in the sample. Although no deadline was set for their return, it was requested that completed questionnaires be returned by April 1, 1967, if possible. Stamped self-addressed envelopes were supplied to encourage a prompt reply. By April 1, seventy questionnaires had been returned. In all, a total of eighty-two questionnaires were returned.

The teacher questionnaires, accompanied by stamped, self-addressed envelopes, were distributed on June 8, 1967. All of the completed teacher questionnaires were received by July 5, 1967.

VI. STATEMENT OF HYPOTHESES TESTED

The following hypotheses were tested:

1. There are no significant differences among the mean attitude scores obtained by calculus, matrices, probability, and control students.
2. Within each of the calculus, matrices, probability, and control groups, there are no significant differences

among the mean attitude scores obtained by very high, high, and moderate ability students.

3. There are no significant differences among the mean anxiety scores obtained by calculus, matrices, probability, and control students.
4. Within each of the calculus, matrices, probability, and control groups, there are no significant differences among the mean anxiety scores obtained by very high, high, and moderate ability students.
5. Within each of the calculus, probability, and matrices groups there are no significant differences among the mean achievement scores obtained by very high, high, moderate ability students on the respective half-course final tests.
6. For each of the calculus, matrices, and probability groups, there is a significant positive relationship between achievement on the trigonometry final test and achievement on the respective half-course final test.
7. There is no significant difference in the mean rank assigned to the three half-courses by university professors;
   a) when the rankings obtained from Engineering, Physical Sciences, Biological Sciences, Mathematics, Humanities, Social Sciences, and Education are analyzed individually.

b) when the rankings from Engineering, Physical Sciences, Biological Sciences, Mathematics, Humanities, Social Sciences, and Education are combined.

VII. STATISTICAL PROCEDURES

To determine the most appropriate statistical procedures, the investigator found it necessary to perform some preliminary tests. Table II shows the frequency distribution of the SCAT scores for each student participating in the study.

TABLE II

FREQUENCY DISTRIBUTION OF SCAT SCORES

SCAT Score (total 110)	Treatment Group			
	Calculus	Matrices	Probability	Control
100 and over	12	--	11	5
90 - 99	23	16	38	21
80 - 89	28	23	21	34
70 - 79	13	20	7	24
60 - 69	8	4	3	8
50 - 59	1	-	1	1
40 - 49	-	2	1	-
Totals	85	65	82	93

Using a one-way analysis of variance, the investigator tested the hypothesis of no significant differences among the mean ability scores for the four groups, Table III shows the

means and variances of the SCAT scores for the four groups. Table IV shows the summary analysis of variance of the SCAT scores.

TABLE III

MEANS AND VARIANCES OF SCAT SCORES

Group	Mean	Variance	
Calculus	85.67	131.30	
Matrices	81.51	116.72	Grand Mean
Probability	89.33	122.20	84.91
Control	82.70	111.08	

TABLE IV

SUMMARY ANALYSIS OF VARIANCE OF SCAT SCORES

Source	SS	MS	df	F	P
Groups	2857.69	952.56	3	7.92	.00
Error	38616.75	120.30	321		

The hypothesis of no significant differences in mean SCAT scores among the four groups was rejected. Using the Newman-Keuls procedure, tests on differences between ordered ability means was carried out.[30] Table V summarizes these

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[30] B. J. Winer, Statistical Principles in Experimental Design (New York: McGraw-Hill Book Company, Inc., 1962), pp. 80-85, 101-103.

results.

If the observed difference between a pair of ordered means exceeds the critical value given in parentheses then the difference was significant at the .05 level. Inspection of Table V revealed that the mean ability score for the probability group was significantly greater than the mean ability score for any other group. Also, the mean ability score for the calculus group was significantly greater than the mean ability score obtained by the matrices group. No other significant differences in ability existed.

TABLE V

TESTS ON DIFFERENCES BETWEEN ORDERED MEAN SCAT SCORES

Group		Probability	Calculus	Control	Matrices
	Means	89.33	85.67	82.70	81.51
Matrices	81.51	7.82 (4.46)	4.16 (4.04)	1.19 (3.40)	-
Control	82.70	6.63 (4.04)	2.97 (3.40)	-	
Calculus	85.67	3.66 (3.40)	-		

Since the design of the experiment called for three ability levels, the investigator attempted to divide the

frequency distribution of the SCAT scores into thirds. It was hoped that this procedure would also divide each group into approximate thirds. This met with reasonable success except for the probability group. For the purpose of this experiment, the three levels of ability were labelled very high, high, and moderate. Table VI shows the number of students in the respective cells.

TABLE VI

CELL FREQUENCIES FOR INSTRUCTIONAL GROUP AND SCHOLASTIC ABILITY LEVEL CLASSIFICATIONS

Group	SCAT Ability Level			Total
	Very High (91 and up)	High (81 - 90)	Moderate (80 and less)	
Calculus	31	31	23	85
Matrices	13	24	28	65
Probability	45	21	16	82
Control	21	37	35	93
Totals	110	113	102	325

A chi-square test carried out on the cell frequencies indicated that they were significantly different from being proportional to the row and column totals.

On the basis of these preliminary tests, it was decided to test hypothesis one and three using a one-way anal-

ysis of covariance method as outlined by Winer.[31] Under this procedure, the criterion scores (the attitude and anxiety scores) are adjusted for the differences the groups showed in ability. The ability score is then termed the covariate. The adjusted mean anxiety and mean attitude scores were then tested for significant differences using the procedure outlined by Winer.[32]

Hypothesis two and four were tested using a one-way analysis of variance statistical procedure.[33] Wherever significant differences were found using this approach, the Newman-Keuls procedure was used to make comparisons between ordered treatment means. In those instances where results were borderline, a t-test was employed to take into account the unequal numbers within the groups. Generally, the Newman-Keuls procedure is more conservative (gives fewer significant results) than the t-test.

One assumption underlying the analysis of variance design is that the variances in the population from which the samples are drawn are equal. Throughout the analyses,

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[31] Ibid., pp. 578-594.

[32] Ibid., pp. 585-586.

[33] Ibid., pp. 46-103.

Bartlett's method was used to test this assumption.[34] One advantage in using analysis of variance is that reasonable departures from this assumption do not seriously affect the validity of the inference drawn from the data.

Hypothesis five could not be tested using an analysis of variance since the assumption of homogeneity of variance was not met. The t'-test statistical procedure, which makes no assumption about the equality of population variance, was used instead. The Welch approximation to sampling distribution of the t' statistic was used since it is the soundest.[35]

Hypothesis six was tested by computing Pearson product-moment correlations between the two sets of raw scores and then testing whether the obtained coefficient differed significantly from zero. Since there were three experimental groups, three product-moment correlations were calculated. Tests were also conducted to determine whether the obtained coefficients differed significantly from one another.[36]

The last hypothesis was tested by using an analysis

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[34] Ibid., p. 95.

[35] Ibid., pp. 36-39.

[36] George A. Ferguson, Statistical Analysis in Psychology and Education (New York: McGraw-Hill Company Inc., 1966), pp. 186-188.

of variance for ranked data as described in Winer.[37] Rather than an F-ratio test, the chi-square statistical procedure is used. This procedure is used to test the hypothesis of no significant difference in mean rank for the treatment groups. In effect, this test is the same as the well-known Friedman test for ranked data except that it may be used whether or not tied ranks are present. Since tied ranks did appear in the data, the Friedman test was not used.

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[37]Winer, op. cit., pp. 136-137.

CHAPTER IV

FINDINGS OF THE STUDY

I. TESTING OF HYPOTHESES

The hypotheses that were stated earlier were tested using the methods outlined in the previous chapter. In this chapter, each of the hypotheses tested is stated immediately before the presentation of the results relevant to that hypothesis. A brief interpretation is given following each set of results. The level of significance used in this study was .05.

Hypothesis One

There are no significant differences among the mean attitude scores obtained by calculus, matrices, probability, and control students.

This hypothesis was tested using an analysis of covariance procedure in which the mean attitude scores for each group were adjusted for the differences in ability between groups. Table VII summarizes the results. Inspection of the probability level leads to rejection of hypothesis one. There were significant differences among the mean attitude scores obtained by the four groups.

TABLE VII

SUMMARY OF ANALYSIS OF COVARIANCE OF ATTITUDE SCORES

Source	df	MS	ADJ F	P
Group	3	725.36	9.19	0.00
Within	320	78.96		

Table VIII shows the adjusted and unadjusted mean attitude scores for the groups.

TABLE VIII

ADJUSTED AND UNADJUSTED MEAN ATTITUDE SCORES

Group	Unadjusted Mean	Adjusted Mean
Calculus	49.96	49.94
Matrices	54.32	54.43
Probability	46.63	46.48
Control	50.00	50.01

Tests were then conducted to determine if the adjusted means were significantly different from one another. In those instances where the results were borderline, a t-test was also used. The significant differences that were found are given in Table IX. The cells with asterisks indicate that the corresponding differences of means are significant at the .05 level. Inspection of Tables VIII and IX

TABLE IX

SUMMARY OF SIGNIFICANT DIFFERENCES AMONG ADJUSTED MEAN ATTITUDE SCORES

Group	Matrices	Control	Calculus
Probability	*	*	*
Calculus	*		
Control	*		

reveal that the matrices group showed a significantly greater mean attitude score than any other group, while the probability group scored significantly lower than any other group.

<u>Hypothesis Two</u>

Within each of the calculus, matrices, probability, and control groups, there are no significant differences among the mean attitude scores obtained by very high, high, and moderate ability students.

Hypothesis two was tested by using the analysis of variance statistical procedure. No significant F-ratios were found for any of the groups and, therefore, this hypothesis was accepted. The mean attitude scores for each ability level within each of the groups are reported in Table X and graphed in Figure 1.

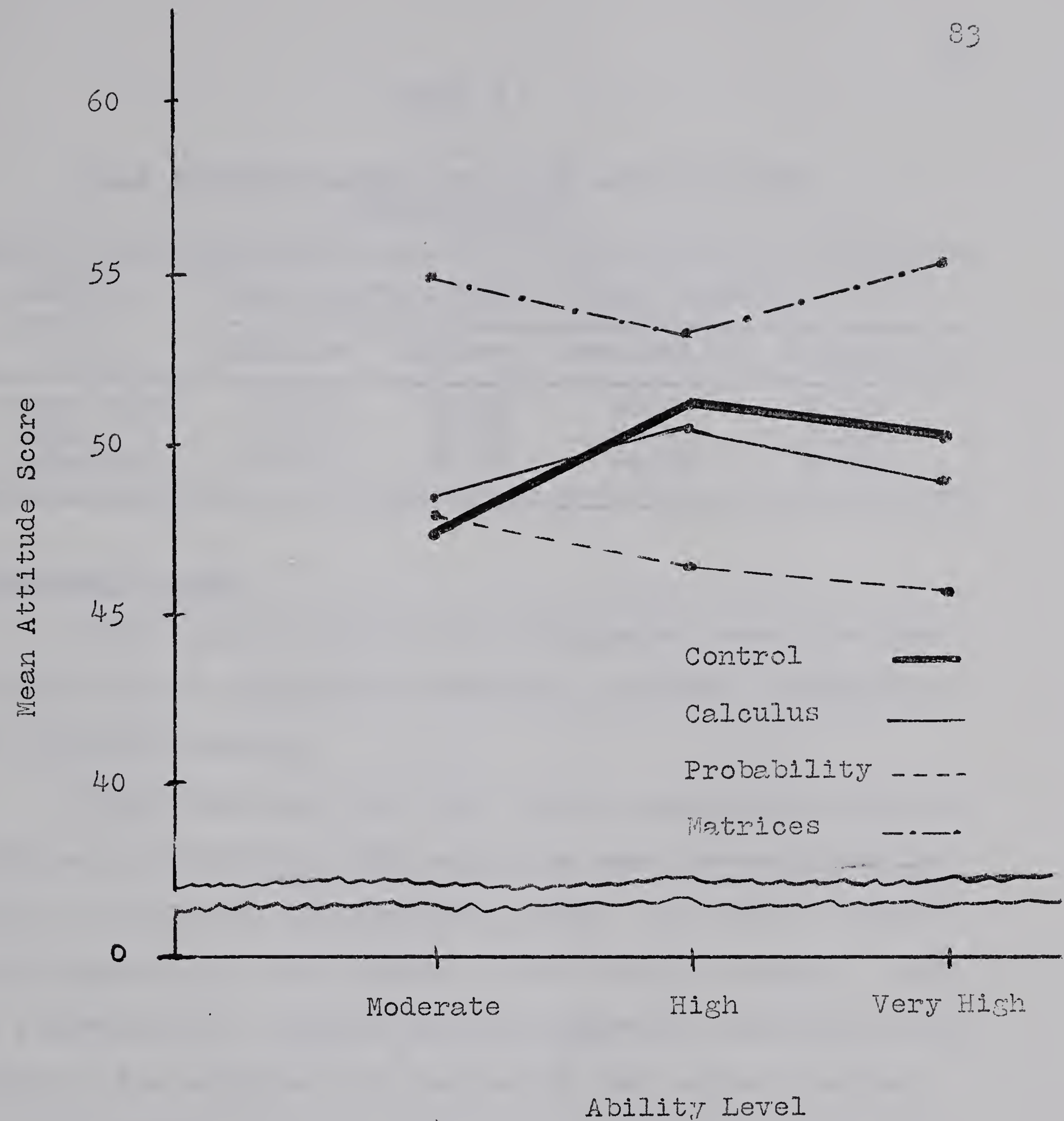

FIGURE 1

MEAN ATTITUDE SCORES FOR ABILITY LEVELS WITHIN EACH GROUP

TABLE X

MEAN ATTITUDE SCORES FOR EACH ABILITY LEVEL WITHIN GROUPS

Ability Range	Mean Attitude Score (total possible 75)			
	Calculus	Matrices	Probability	Control
Very High	49.45	55.38	46.04	50.62
High	51.03	53.46	46.43	51.30
Moderate	49.21	54.57	48.56	48.26

Hypothesis Three

There are no significant differences among the mean anxiety scores obtained by calculus, matrices, probability, and control students.

This hypothesis was also tested using an analysis of covariance procedure. Following the same precedure as that used for analyzing the attitude scores, the anxiety scores were adjusted for the effects of the ability scores. Table XI summarizes the results of this procedure while Table XII contains the adjusted and unadjusted mean anxiety scores for the groups.

All possible pairs of adjusted means were tested to determine if they were significantly different. Table XIII gives the results of these tests.

TABLE XI

SUMMARY OF ANALYSIS OF COVARIANCE OF ANXIETY SCORES

Source	df	MS	ADJ F	P
Group	3	614.31	14.34	0.00
Within	320	42.85		

TABLE XII

ADJUSTED AND UNADJUSTED MEAN ANXIETY SCORES

Group	Unadjusted Mean	Adjusted Mean
Calculus	32.54	32.66
Matrices	27.94	27.41
Probability	28.43	29.13
Control	33.63	33.29

TABLE XIII

SUMMARY OF SIGNIFICANT DIFFERENCES AMONG ADJUSTED MEAN ANXIETY SCORES

Group	Control	Calculus	Probability
Matrices	*	*	
Probability	*	*	
Calculus			

Inspection of Tables XII and XIII show that both the control group and the calculus group showed higher mean anxiety scores than either the matrices or the probability groups. No significant difference in mean anxiety existed between the calculus and control groups. Similarly, no significant difference in mean anxiety was found between the matrices and probability groups. Thus, hypothesis three was rejected.

<u>Hypothesis Four</u>

Within each of the calculus, matrices probability, and control groups, there are no significant differences among the mean anxiety scores obtained by very high, high, and moderate ability students.

Using an analysis of variance approach, the above hypothesis was tested. The F-ratios and probability level obtained from these analyses are shown in Table XIV.

TABLE XIV

SUMMARY OF F-RATIOS AND PROBABILITY LEVELS FOR HYPOTHESIS FOUR

Group	F	P
Calculus	3.70	0.03
Matrices	0.01	0.99
Probability	1.14	0.32
Control	10.57	0.00

Since the F-ratios for the calculus and control groups exceeded the critical value required for statistical significance, the results of the analyses for these two groups is reported below. Table XV illustrates the results of the analysis of variance performed on the data from the calculus group.

TABLE XV

SUMMARY OF ANALYSIS OF VARIANCE OF ANXIETY SCORES FOR ABILITY LEVELS WITHIN CALCULUS GROUP

Source	SS	MS	df	F	P
Groups	378.85	189.42	2	3.70	0.03
Error	4200.20	51.22	82		

With the calculus group, there were significant differences among the mean anxiety scores obtained by very high, high, and moderate ability students.

Following the significant F-value, the Newman-Keuls method was used to make comparisons between all pairs of ordered means. The results of this procedure are found in Table XVI. The numbers in parentheses indicate the critical value that must be exceeded for significance at the .05 level. The moderate ability students showed a significantly higher anxiety mean than students in either the high or very high

ability levels.

TABLE XVI

TESTS ON DIFFERENCES BETWEEN ORDERED ANXIETY MEANS FOR ABILITY LEVELS WITHIN CALCULUS GROUP

Ability Level		Moderate	Very High	High
	Means	36.00	31.42	31.09
High	31.09	4.90 (4.60)	0.32 (3.84)	-
Very High	31.42	4.58 (3.84)	-	
Moderate	36.00	-		

A summary of the analysis of variance performed on the anxiety scores for the control group is given in Table XVII, while Table XVIII contains the corresponding Newman-Keuls results.

TABLE XVII

SUMMARY OF ANALYSIS OF VARIANCE OF ANXIETY SCORES FOR ABILITY LEVELS WITHIN CONTROL GROUP

Source	SS	MS	df	F	P
Groups	814.27	407.13	2	10.57	0.00
Error	3467.30	38.53	90		

TABLE XVIII

TESTS ON DIFFERENCES BETWEEN ORDERED ANXIETY MEANS FOR ABILITY LEVELS WITHIN CONTROL GROUP

Ability Level		Moderate	High	Very High
	Means	37.43	31.60	30.91
Very High	30.91	6.52 (3.89)	0.69 (3.12)	-
High	31.60	5.83 (3.12)	-	
Moderate	37.43	-		

Within the control group, there were significant differences among the mean anxiety scores obtained by very high, high, and moderate ability students. The moderate ability students showed a significantly higher mean anxiety score than either the high or very high ability students within the control group.

The respective mean anxiety scores for each ability level within each of the four groups are displayed in Table XIX. These mean scores are also illustrated in Figure 2.

Summary of results for hypothesis four. Analysis of the data revealed that for two of the groups, hypothesis four was rejected. In both the calculus and control groups, the moderate ability students showed significantly greater an-

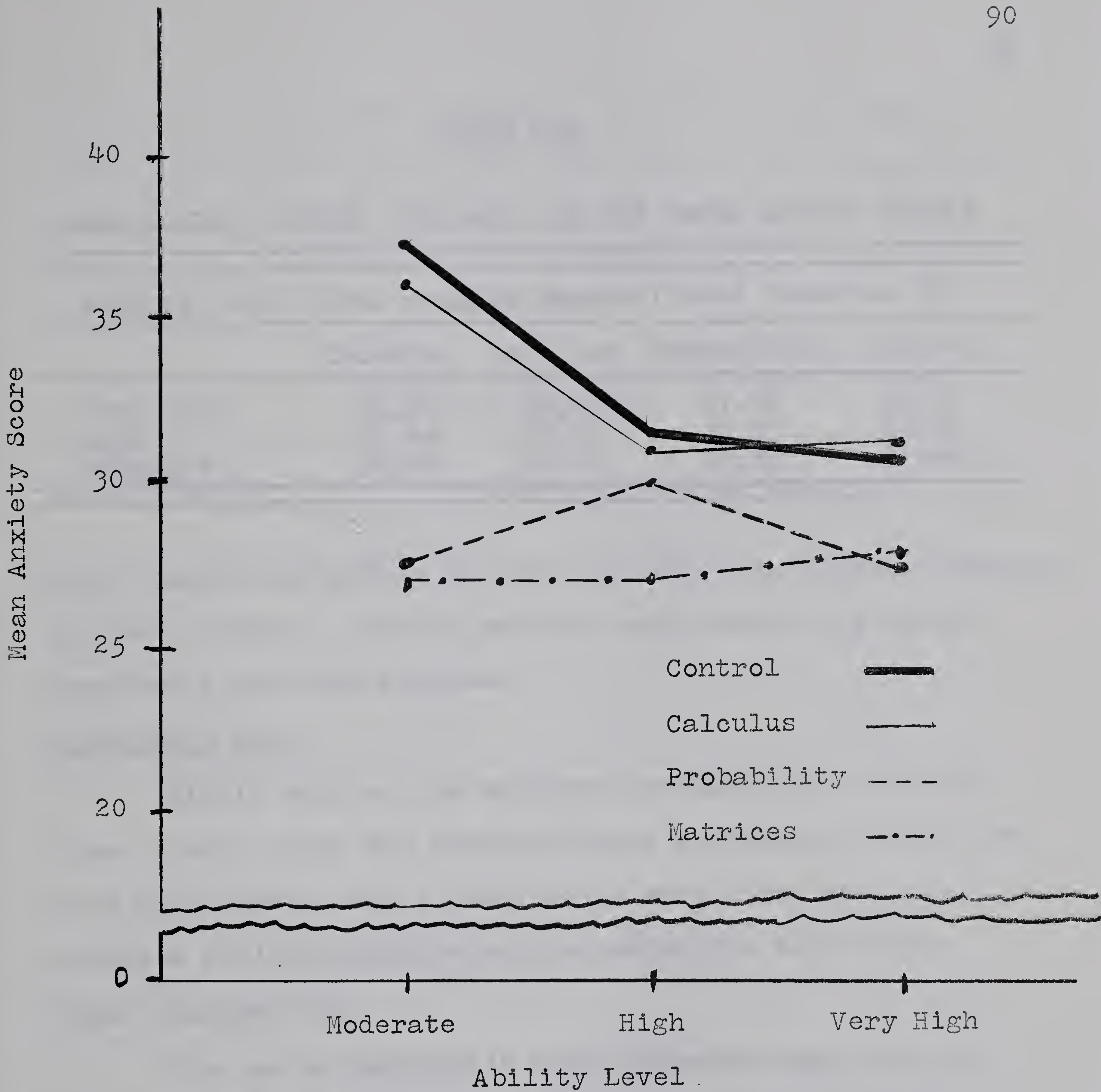

FIGURE 2

MEAN ANXIETY SCORES FOR ABILITY LEVELS WITHIN EACH GROUP

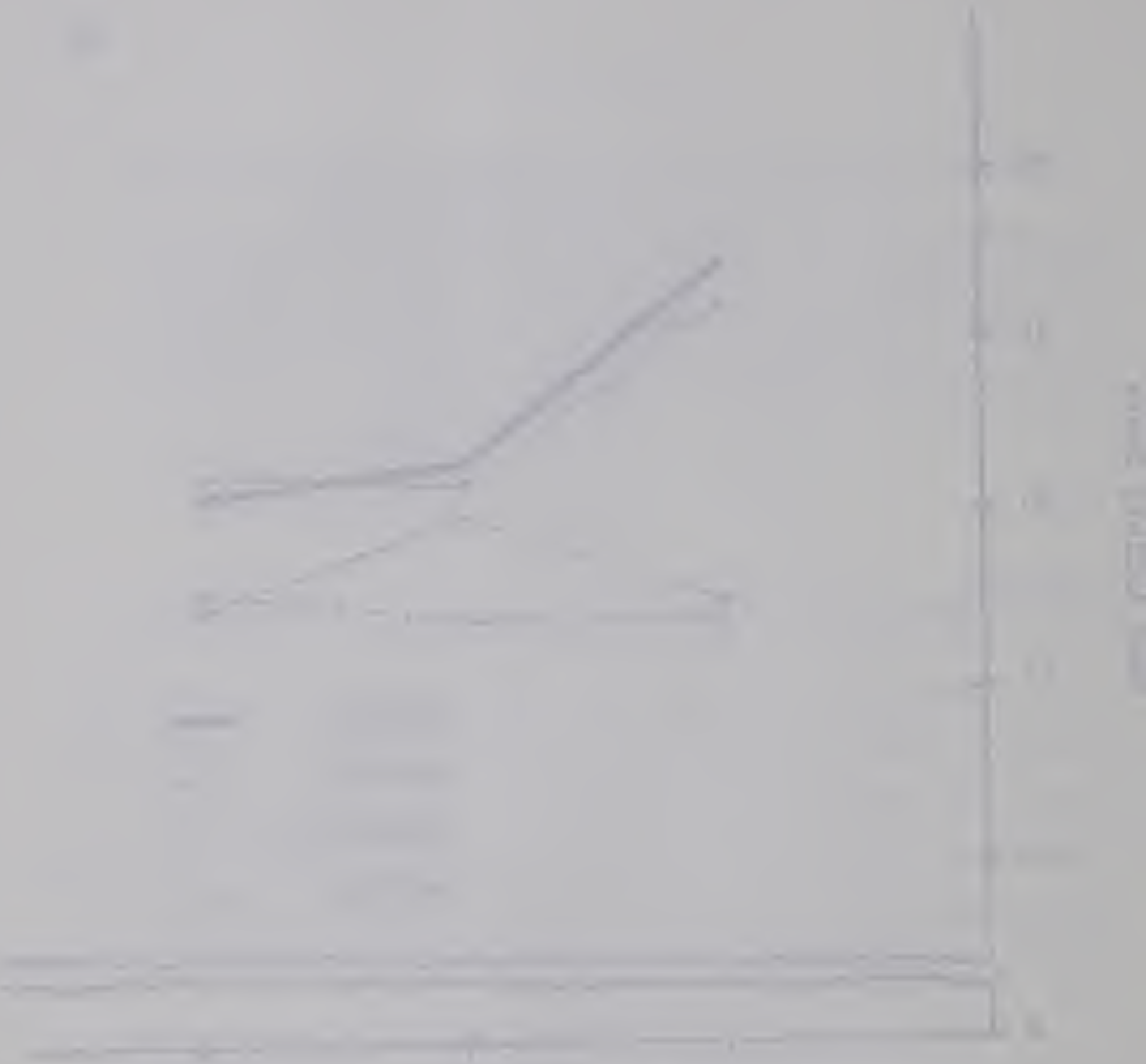

TABLE XIX

MEAN ANXIETY SCORES FOR EACH ABILITY LEVEL WITHIN GROUPS

Ability Level	Mean Anxiety Scores (total possible 60)			
	Calculus	Matrices	Probability	Control
Very High	31.42	28.08	27.76	30.91
High	31.09	27.79	30.05	31.60
Moderate	36.00	28.00	28.25	37.43

xiety means than either the high or very high ability students in these groups. For the matrices and probability groups, hypothesis four was accepted.

Hypothesis Five

Within each of the calculus, probability, and matrices groups there are no significant differences among the mean achievement scores obtained by very high, high, and moderate ability students on the respective half-course final examinations.

The use of Bartlett's test indicated that for all three groups, the assumption of homogeneity of variance of achievement scores for each ability level within the groups was not met. Instead of using the analysis of variance statistical procedure, the investigator used the t'-test procedure to test differences between pairs of means.

Tables XX and XXI show the means and variances of the half-course achievement scores for each ability level within the groups.

TABLE XX

HALF-COURSE ACHIEVEMENT SCORE MEANS FOR EACH ABILITY LEVEL WITHIN GROUPS

Ability Level	Mean Achievement Score		
	Calculus	Matrices	Probability
Very High	67.16	52.69	55.07
High	60.68	46.38	51.14
Moderate	53.17	34.79	48.06
Grand Mean	61.01	42.63	52.70
Total Possible	86.00	59.00	67.00

TABLE XXI

HALF-COURSE ACHIEVEMENT SCORE VARIANCES FOR EACH ABILITY LEVEL WITHIN GROUPS

Ability Level	Variance		
	Calculus	Matrices	Probability
Very High	138.21	24.40	81.38
High	350.23	87.38	78.03
Moderate	203.51	142.23	23.00

The Welch test, as described earlier, was then employed to test all parts of hypothesis five using the means

shown in Table XX. The outcome of the Welch test for each of the experimental groups is summarized in Tables XXII, XXIII, and XXIV respectively. The entries in the tables are the observed t' values and the asterisks indicate a significant difference at the .05 level.

TABLE XXII

WELCH TEST RESULTS OF DIFFERENCES BETWEEN PAIRS OF HALF-COURSE ACHIEVEMENT SCORE MEANS FOR ABILITY LEVELS WITHIN CALCULUS GROUP

Ability Level	High	Moderate
Very High	1.63	3.86*
High	-	1.67

TABLE XXIII

WELCH TEST RESULTS OF DIFFERENCES BETWEEN PAIRS OF HALF-COURSE ACHIEVEMENT SCORE MEANS FOR ABILITY LEVELS WITHIN MATRICES GROUP

Ability Level	High	Moderate
Very High	2.69*	6.79*
High	-	3.93*

<u>Summary of results for hypothesis five</u>. Hypothesis five was rejected. For the matrices group, the very high ability students showed a significantly greater mean achieve-

TABLE XXIV

WELCH TEST RESULTS OF DIFFERENCES BETWEEN PAIRS OF HALF-COURSE ACHIEVEMENT SCORE MEANS FOR ABILITY LEVELS WITHIN PROBABILITY GROUP

Ability Level	High	Moderate
Very High	1.67	3.89*
High	-	1.36

ment score than either the high or moderate ability students. Also, the high ability students demonstrated a significantly higher achievement level than the moderate ability students. Within each of the calculus and probability groups, the very high ability students obtained a significantly greater mean achievement score that did the moderate ability students. No other significant differences were found within the calculus and probability groups respectively.

Hypothesis Six

For each of the calculus, matrices, and probability groups there is a significant positive relationship between achievement on the trigonometry final examination and achievement on the respective half-course final examination.

Pearson product-moment correlations were calculated between these two variables using the raw scores from each test. The correlations obtained from each group are entered

in Table XXV.

TABLE XXV

PEARSON PRODUCT-MOMENT CORRELATIONS BETWEEN TRIGONOMETRY ACHIEVEMENT SCORES AND HALF-COURSE ACHIEVEMENT SCORES FOR EACH EXPERIMENTAL GROUP

Group	Calculus	Matrices	Probability
Pearson r	0.74*	0.78*	0.68*

The asterisks indicate that the reported correlations are significantly different from zero at the .05 level; therefore, hypothesis six was accepted. Using Fisher's z_r transformation, tests were conducted to determine if these correlations differed significantly from each other. No significant differences were found.

<u>Hypothesis Seven</u>

There is no significant difference in mean rank assigned to the three half-courses by university professors:

a) when the rankings obtained from Engineering, Physical Science, Biological Sciences, Mathematics, Humanities, Social Sciences, and Education are analyzed individually.

b) when the rankings obtained from Engineering, Physical Science, Biological Sciences, Mathematics, Humanities, Social Sciences, and Education are combined.

Eighty-two of the university questionnaires that were sent out were returned and sixty-four of the respondents ranked the half-courses while the remaining eighteen stated no preference. Table XXVI gives the breakdown of the returned questionnaires.

Hypothesis 7a was then tested using the analysis of variance procedure for ranked data as outlined in the previous chapter. Table XXVII shows the results of these analyses. Where tied ranks occurred, the tied values were replaced by the average rank. Only the sixty-four questionnaires that showed ranking for the half-courses were analyzed.

TABLE XXVII

ANALYSIS OF RANKS ASSIGNED TO THE THREE HALF-COURSES FOR EACH OF THE SEVEN MAJOR AREAS OF INSTRUCTION

Major Area of Instruction	Sum of Ranks			X^2_{ranks}
	Calculus	Matrices	Probability	
Engineering	13	27	38	24.15*
Physical Sciences	13	34.5	30.5	20.51*
Biological Sciences	19.5	25.5	9	16.91*
Mathematics	20	27	31	4.97
Humanities	7.5	6	4.5	1.64
Social Sciences	11	12	7	2.80
Education	19	19	10	6.75*
Total	103	131	151	

TABLE XXVI

BREAKDOWN OF RETURNED UNIVERSITY QUESTIONNAIRES

Major Area of Instruction	Number of Professors Ranking Courses	Number of Professors Stating No Preference	Total
Engineering	13	-	13
Physical Sciences	13	-	13
Biological Sciences	9	-	9
Mathematics	13	-	13
Humanities	3	9	12
Social Sciences	5	5	10
Education	8	4	12
Total	64	18	82

Hypothesis 7a was rejected for four of the seven areas. The asterisks in Table XXVII reveal that for the areas of Engineering, Physical Sciences, Biological Sciences, and Education, the hypothesis of no difference in the assigned mean rank for the half-courses was rejected. For the remaining three areas hypothesis 7a was accepted.

All the major instructional areas were combined to test hypothesis 7b. A summary of the analysis of variance for this data is given in Table XXVIII.

TABLE XXVIII

SUMMARY OF ANALYSIS OF RANKS ASSIGNED TO THE THREE HALF-COURSES WHEN SEVEN MAJOR INSTRUCTIONAL AREAS ARE COMBINED

Sum of Ranks			Source	SS	df	X^2_{ranks}
Calculus	Matrices	Probability				
103	131	151	Treatment	18.09	2	18.60*
			Within People	124.50		

Hypothesis 7b was rejected. The asterisk in Table XXVIII reveals that there was a significant difference in the mean rank assigned to the three half-courses. The order of preference was calculus, matrices, and then probability.

The second major question on the university question-

naire asked the professors whether or not they thought it essential for freshman students to have a knowledge of the half-course that they ranked number one before the students entered university. An affirmative response was given by twenty professors, thirty-seven professors answered negatively, while seven stated that this knowledge was highly desirable but not essential. Of the twenty that indicated the knowledge of the half-course was essential, seventeen had given the highest rank to calculus. Some of the comments that the professors made regarding the half-courses are found in Appendix I.

II. INTERRELATIONSHIP AMONG VARIABLES

This study provided an excellent opportunity to report the interrelationships among the variables considered in this project. For each of the experimental groups, measures of attitude, anxiety, half-course achievement, trigonometry achievement, and ability were available. A matrix showing the Pearson product-moment correlations among these variables was prepared for each experimental group. These appear in Tables XXIX to XXXI. The asterisks indicate significance at the .05 level.

TABLE XXIX

ATTITUDE, ANXIETY, HALF-COURSE ACHIEVEMENT, TRIGONOMETRY ACHIEVEMENT, AND ABILITY SCORE CORRELATIONS FOR CALCULUS GROUP

Variable	Anxiety	Achievement (Half-Course)	Ach. (Trig.)	Ability
Attitude	-0.40*	0.19*	0.25*	-0.00
Anxiety		-0.50*	-0.53*	-0.28*
Achievement (Half-Course)			0.74*	0.36*
Ach. (Trig.)				0.50*

TABLE XXX

ATTITUDE, ANXIETY, HALF-COURSE ACHIEVEMENT, TRIGONOMETRY ACHIEVEMENT, AND ABILITY SCORE CORRELATIONS FOR MATRICES GROUP

Variable	Anxiety	Achievement (Half-Course)	Ach. (Trig.)	Ability
Attitude	-0.23	0.27	0.17	-0.03
Anxiety		-0.19	-0.24	-0.09
Achievement (Half-Course)			0.78*	0.68*
Ach. (Trig.)				0.60*

TABLE XXXI

ATTITUDE, ANXIETY, HALF-COURSE ACHIEVEMENT, TRIGONOMETRY ACHIEVEMENT, AND ABILITY SCORE CORRELATIONS FOR PROBABILITY GROUP

Variable	Anxiety	Achievement (Half-Course)	Ach. (Trig.)	Ability
Attitude	-0.10	0.32*	0.14	0.02
Anxiety		-0.35*	-0.39*	-0.14
Achievement (Half-Course)			0.68*	0.38*
Ach. (Trig.)				0.36*

III. TEACHER QUESTIONNAIRE FINDINGS

The teacher questionnaire had a two-fold purpose: to obtain information on the characteristics of the teachers involved in the Mathematics 31X program and to determine the teachers' evaluation of the respective half-course. A summary of the information gathered from the teachers follows.

Teacher Characteristics

The success of any experimental program depends largely upon the teachers involved. Although the number of years of university training and the number of years of teaching experience do not imply that good teaching is taking place, it is useful to compare this information for the three experimental groups. Table XXXII contains much of this material and Table XXXIII summarizes it further. This in-

TABLE XXXII

SELECTED CHARACTERISTICS OF TEACHERS INVOLVED IN MATHEMATICS 31X PROGRAM

Characteristic	Calculus Teachers					Matrices Teachers					Probability Teachers			
	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Number of years of university training	6	6	4	5	4	5	6	8	5	4	6	6	4	6
Number of university degrees held	2	2	1	1	1	2	2	3	1	1	2	2	1	2
Number of years of teaching experience	18	8	39	17	36	17	22	19	18	6	23	34	35	33
Number of years of teaching high school mathematics	12	8	26	17	5	17	18	13	13	2	23	20	23	31
Number of university mathematics courses in which credit was received	10	9	5	6	2	7	12	5	8	4	5	5	3	5
Indicated that mathematics was teaching major		X	X	X		X	X	X	X		X			X

TABLE XXXIII

AVERAGE HALF-COURSE INSTRUCTIONAL TIME, AVERAGE NUMBER OF STUDENTS PER CLASS, AND AVERAGES OF SOME TEACHER CHARACTERISTICS FOR EACH GROUP OF TEACHERS

	Calculus	Matrices	Probability
Average time spent in half-course instruction in minutes per week	204	207	206
Average number of students per class enrolled in half-course	19	21	26
Average number of years of university training per teacher	5.0	5.6	5.5
Average number of years of teaching experience per teacher	23.6	19.4	31.3
Average number of years of high school mathematics teaching experience per teacher	15.6	12.6	24.3
Average number of university mathematics courses taken per teacher	6.4	7.2	4.5

formation appeared in part five of the questionnaire.

Format of the Textbook

Part one of the teacher questionnaire pertained to the format of the respective textbooks presecribed for the experimental half-courses. The teachers were asked to rate the textbook as excellent, good, fair, or poor on the nine stated features. Table XXXIV shows the ratings indicated by each teacher involved in the program. The entries on the table are to be interpreted as follows:

E--excellent

G--good

F--fair

P--poor

The Pupil and the Half-Course

Part two of the teacher questionnaire contained thirteen short answer questions that were designed to determine how well each half-course related to the pupils involved. Rather than tabulate the response given by each teacher to every question, this information is summarized in Table XXXV. The entries in the table indicate the consensus of opinion of the teachers from each of the three groups. In those instances where the teachers could not reach a mutual agreement, the phrase "no consensus" appears.

Comments Pertaining to Table XXXV

There were two questions for which the teacher re-

TABLE XXXIV

TEACHER RATINGS ON THE FORMAT OF THE TEXTBOOK

Textbook Feature	Calculus Teachers					Matrices Teachers					Probability Teachers			
	A	B	C	D	E	F	G	H	I	J	K	L	M	N
The effectiveness of diagrams, graphs, illustrations, etc.	F	F	G	F	F	G	F	F	F	F	G	G	G	G
The number of exercises provided	F	P	F	P	F	F	G	G	G	G	G	G	F	G
The variety of exercises provided (skill, application, comprehension, synthesis)	F	G	G	P	F	G	F	G	G	G	G	F	G	G
The range of difficulty of exercises provided	G	G	F	P	F	G	G	F	F	G	G	G	F	G
The number of sample problems provided	F	P	F	P	P	F	P	P	F	F	P	G	G	G
The effectiveness of the written presentations for Mathematics 31 students	G	P	G	F	P	F	G	F	G	F	P	G	G	G
The usefulness of the text for individual study by the student	F	P	F	F	P	P	F	P	G	F	P	G	G	G
The usefulness of the text as a basis for classroom discussion	G	F	E	F	F	F	F	F	G	G	F	G	G	G
The overall appeal of the textbook format	F	F	G	F	P	F	F	F	G	F	F	G	G	G

sponses showed considerable variation. These pertained to student attitude and to the extent to which the half-course was meeting the mathematical needs of the students.

Two of the calculus teachers indicated that the students displayed less favorable attitudes towards the half-course as compared to the attitudes shown toward trigonometry. Another two said that the students' attitudes towards the half-course were more favorable than they were towards trigonometry. The fifth teacher indicated that the attitudes displayed were about the same. For the probability group, one teacher indicated that the students' attitudes were more favorable, another indicated less favorable, the third indicated no difference in expressed attitude, and the fourth stated that he had no idea of how the students' attitudes compared on the different material.

With regard to the extent to which the half-courses met the mathematical needs of the students when compared to trigonometry, two of the calculus teachers replied that the half-course met these needs "better", two indicated "poorer", and the fifth remained neutral on this point. For the matrices group, two teachers indicated "better", one indicated "poorer", and the other two responded "the same". Of the four probability teachers, two indicated "the same" and two indicated "poorer".

TABLE XXXV

SUMMARY OF TEACHER OPINIONS ON THE RELATIONSHIP OF THE HALF-COURSE TO THE PUPIL

Question	Calculus	Matrices	Probability
The length of the half-course	adequate	adequate	adequate
The difficulty of the concepts in the half-course compared to the difficulty of the concepts in trigonometry	more difficult	the same	the same
The students' attitude towards the half-course as compared to the students' attitude toward trigonometry	no consensus	more favorable	no consensus
The mathematical value of the half-course for a grade twelve matriculation student who is mathematically inclined	very good	good	good
Compared to trigonometry, the emphasis placed upon computational skills in the half-course	less	less	less
Compared to trigonometry, the emphasis placed upon the understanding of concepts and principles in the half-course	no consensus	greater	greater
Compared to trigonometry, the emphasis placed upon terminology in the half-course	greater	the same	greater
Compared to trigonometry, the emphasis placed upon symbolism in the half-course	greater	the same	greater
Compared to trigonometry the concepts in the half-course require the students to think	much more	more	more

TABLE XXXV (Con't)

Question	Calculus	Matrices	Probability
Compared to trigonometry, how well do you feel that the half-course is meeting the mathematical needs of your students	no consensus	no consensus	no consensus
If a student is to benefit by taking the half-course, a "B" or better in Mathematics 20 is	essential	essential	essential
If a student is to benefit by taking the half-course, Mathematics 30	should be taken concurrently	should be taken concurrently	no consensus
If a student is to benefit by taking the half-course, trigonometry	should be a prerequisite	should be taken concurrently	is not required

Comments on the General Impression of Each Half-Course

Each statement below represents one teacher's comment pertaining to the above sub-heading.

Calculus teachers.

1. Very suitable for matriculating students if going into university mathematics.

2. Good. I think the course should be continued as a half-course, along with other half-courses in mathematics.

3. Much of the material should be covered in the new Mathematics 31 if properly taught.

4. It could be a good introduction to the calculus of first year university.

5. Good idea!

Matrices teachers.

1. If there were more time available, we could accomplish much more. The year ran out as we were getting into the real "meat" of the course. I feel that on a second round, we would progress more rapidly.

2. I feel that the linear algebra ideas along with their application cannot be properly studied in the time allotted. I feel that a full course in this subject would give the student a better chance to grasp the significance of the subject.

3. A good course--students enjoyed it. It was like a breath of spring air to the class--something new--stimulating and refreshing. Allowed even some students who are low achievers because of lack of background in conventional mathematics to do well since background requirements are less than that for algebra.

4. Good basic material for students likely to enter university. Otherwise of little value.

5. By and large favorable. Students found the existence of a non-commutative system intriguing. I felt they learned more mathematics--the attitude was more positive than for trigonometry. The reason may have been the "Hawthorne Effect".

Probability teachers.

1. A worthwhile course but not worth the time in the future because all but the last two chapters are repetitious with the new Mathematics 30 course.

2. This is good. The students who have studied the new mathematics in Mathematics 20 would be able to cover this course in much less time as they are familiar with the terminology. Even these students could have done the work in four months.

3. What does it lead to? It was fun. I liked it!

4. Interesting, but not suitable as a half-course because it repeats too much of the new Mathematics 30. We avoided most of the difficult topics but with our treatment of the course, I can see where many of these could have been attempted.

Comments on the Prescribed Text for Each Half-Course

Calculus teachers.

1. Rather restricted in variety of exercises. The jump is rapid from easy to difficult exercises.

2. Too much theory, not enough questions, and not enough "practical" problems.

3. The text is not geared to the grade twelve level.

4. Not too bad--short on exercises and examples.

5. Suitable text. Solid mathematic approach. Some questions at end are rather vague and difficult.

Matrices teachers.

1. The textual materials were somewhat rigorous in approach. Considerable time is spent in arithmetic computation. I would have preferred to have extra time for applications.

2. Improves as one gets into it. Written for superior students (at grade twelve level of reference).

3. Fair. Too much time spent on rigorous proofs---the abstract proofs. But the proofs can be left out and the course taught without them. This makes a better course.

4. The text, I feel, is a good one provided it is covered completely so that the applications of the theory could be better demonstrated. As it turned out, time did not permit study of the linear algebra applications in vectors or the theory and use of determinants.

5. This text is used at university level and is less suitable for high school students than others I have received.

Probability teachers.

1. On many topics it could be more clearly written. Some terms are not defined. A few questions involve theory which is not presented. However, it seems to be the most suitable of the texts available at the high school level.

2. Good.

3. The text is good but we had to use other books for exercises to get variety and more challenging questions.

4. Good. The treatment is much better than that in the new Mathematics 30 course.

Comments on Whether the Pupil Growth in Mathematics was Sufficient to Justify Teaching the Half-Course to Mathematics 31 Students

Calculus teachers.

1. Growth sufficient except in one unique instance. One student was taking Mathematics 20, 30, and 31X. His background in mathematics was weak.

2. Yes!

3. Only the top students seemed to feel at home with the course.

4. Yes!

5. Yes, especially those coming from the traditional Mathematics 30 course.

Matrices teachers.

1. Yes--definitely.

2. Yes.

3. Yes.

4. Yes, for the more able students, about two-thirds of my class.

5. Yes, though it should be remembered that I had a group that was very much selected. In several instances, students found linear algebra much more interesting than trigonometry.

Probability teachers.

1. This year, yes. In future, no.

2. The half-course in probability is only an introduction to another phase of mathematics. I think it worthwhile.

3. Yes, but there are too many areas of study for high school students. The average and below average student gets "mowed under" by the multiplicity of content in various subject areas.

4. The amount of new mathematics introduced, considering the content of the new Mathematics 30, is very small and would not justify a half-course in probability.

Comments on the Coherence of Each Half-Course

Calculus teachers. Nearly all of the calculus teachers reported that they felt the half-course was sufficiently unified.

Matrices teachers. Generally, the teachers of matrix algebra felt that the half-course was quite coherent. Some commented that more cohesion would have been evident if time had allowed the teacher to dwell more on the applications of matrix algebra to various problems.

Probability teachers. All of the probability teachers felt that the half-course was coherent. Most of them indicated that the coherence of the course became more apparent in the later sections where seemingly disjointed, initial topics were brought together.

Comments on the Use of Teacher Guidebooks

Calculus teachers. Apparently no guidebook accompanied the calculus text since none of the teachers used one. However most of these teachers indicated that a guidebook would have helped them to prepare their lessons.

Matrices teachers. All of the matrices teachers used the guidebook that accompanied the text. On the whole, these teachers found the guidebook quite helpful as a response verification source, but of limited help in presentation of material.

Probability teachers. The guidebook that followed the probability text was used by all four of the probability teachers. Again, most of the teachers used it as a response verification book and according to the teachers, in many

instances the answers given were incorrect. Only one teacher indicated that the guidebook helped him to any great extent.

Comments on the Academic Training of Teachers

With regard to their own academic training, the participating teachers were asked whether or not they felt well-prepared to teach the respective half-courses.

Calculus teachers. Two of the five calculus teachers indicated that they were not well prepared to teach the half-course. Both of these teachers indicated that a recent review of calculus was needed, especially a course emphasizing theoretical calculus.

Matrices teachers. Three of the five matrices teachers reported that their mathematical background was insufficient as far as preparation for teaching matrix algebra was concerned. All of the teachers in this group suggested that the equivalent of a full-year course in matrix algebra would be extremely helpful as further preparation.

Probability teachers. Not one of the probability teachers indicated that he was poorly prepared to teach the course. Two of the teachers thought they were adequately prepared while a third felt that any further preparation that was required could be obtained through home study. The remaining teacher indicated that an evening credit or summer school dealing with probability and related topics would be of assistance.

Comments on Speculative Questions

The fourth section of the teacher questionnaire dealt with questions that required the teachers to extrapolate beyond the experience that they had with the half-course. The questions were designed to seek the opinions and judgments of the Mathematics 31X teachers as guidelines for future revisions of the high school mathematics curriculum. Table XXXVI contains the consensus of opinion of the teachers from each of the experimental groups.

TABLE XXXVI

SUMMARY OF TEACHER OPINION ON SPECULATIVE QUESTIONS RELATIVE TO THE HALF-COURSES

Question	Calculus	Matrices	Probability
The mathematical value of the half-course content for any grade twelve matriculation student	good	fair	good
The mathematical value of the half-course content for any grade twelve student in the vocational or technical pattern	poor	poor	fair
How well do you think the half-course would accomodate students of varying degrees of ability?	poorly	no consensus	adequately
Do you think it would be feasible to teach a six-week unit on the material in the half-course?	no	no	no consensus
Do you think that most high school teachers could teach the half-course adequately to Mathematics 31 students?	no	no	yes
Assuming matriculation-calibre students, at what grade level do you think the half-course could be most profitably taught?	12	12	12
Which of the following arrangements would you favor for the Mathematics 31 course? a. a full year of trigonometry b. the Mathematics 31X course c. other	a. no teacher b. three teachers c. two teachers	a. no teacher b. two teachers c. three teachers	a. one teacher b. one teacher c. one teacher

CHAPTER V

SUMMARY OF RESULTS, LIMITATIONS, CONCLUSIONS, RECOMMENDATIONS, AND IMPLICATIONS FOR FURTHER RESEARCH

I. REVIEW OF THE THESIS

The major aim of this study was to evaluate three mathematics half-courses taught as part of the Mathematics 31X course during the 1966-67 school year in some Alberta high schools. These half-courses, all of an introductory nature, were Calculus, Probability and Statistics, and Matrix Algebra. The total number of students from the experimental groups that participated in the study was 230. A control group numbering ninety-three students also participated in the study. These students studied the normal Mathematics 31 course (plane trigonometry and related topics) during the 1966-67 school year. All of the students involved in the study were enrolled in grade twelve and ranged in age from sixteen to twenty-one years.

Information was gathered from a number of sources to make the evaluation as broad as possible but still manageable. Measures of four student variables were obtained. These variables included attitude, anxiety, achievement, and scholastic ability. Additional information was supplied by teachers participating in the program as well as professors from the University of Alberta.

Seven major hypotheses were drafted and then tested using a variety of suitable statistical procedures. The reader is referred to Chapter III for a detailed explanation of the statistics used. For this experiment a difference of two mean scores were considered statistically significant only if the probability of observing such a difference as a result of sampling error was .05 or less.

II. SUMMARY OF RESULTS

Results from Statistical Analyses

The statistical analyses of the data collected for this study revealed the following results:

1. On the attitude items used in this study:
   a) the students who studied matrix algebra showed a significantly more favorable attitude than those students who had studied calculus, probability, or a full year of trigonometry.
   b) the students who had studied probability showed a significantly less favorable attitude than those students who had studied calculus, matrix algebra, or a full year of trigonometry.
   c) no other significant differences in attitude were found.

2. On the anxiety items used in this study:
   a) the students who studied calculus showed a significantly greater mean anxiety level than those who studied probability or matrix algebra.
   b) the students who studied trigonometry for a full year showed a significantly greater mean anxiety level than those who studied probability or matrix algebra.
   c) no other significant differences in anxiety were found.

3. Within each of the four groups, there were no significant differences among the mean attitude scores obtained by very high, high, and moderate ability students.

4. a) The moderate ability students who studied calculus showed a significantly greater mean anxiety level than either the high or very high ability students who studied the same material.
   b) The moderate ability students who studied a full year of trigonometry showed a significantly greater mean anxiety level than either the high or very high ability students.
   c) No other significant differences in mean anxiety were found among the three different ability levels within each of the four groups.

5. a) For each of the experimental groups, the very high ability students achieved significantly better than the moderate ability students on the respective half-course final test.
   b) For the matrix algebra group, the very high ability students showed a significantly greater mean achievement score than the high ability students. Also, the high ability students showed a significantly greater mean achievement score than the moderate ability students. These latter differences were not found within the calculus, or probability groups.

6. For each of the experimental groups, there was a significant positive relationship between achievement on the trigonometry final test and achievement on each respective half-course final test. The Pearson product-moment correlations found were 0.74, 0.78, and 0.68 for the calculus, matrices, and probability groups respectively.

7. On the basis of the content of the half-courses:
   a) sixty-four university professors ranked a knowledge of calculus as being most beneficial for students entering first year university. Matrix algebra was ranked second and probability ranked third.
   b) a knowledge of calculus was ranked significantly higher than either of the other two half-courses for those students entering engineering or the physical sciences.
   c) a knowledge of probability was ranked significantly higher than either of the other two half-courses for those students entering the biological sciences or education.
   d) no significant difference in the mean rank assigned to the half-courses was found for those students entering mathematics, social sciences or humanities.

<u>Results from Teacher Questionnaire</u>

Examination of the responses to the questions on the teacher questionnaire are summarized as follows:

1. All of the participating teachers reported that their respective half-course was sufficiently unified and of adequate length.

2. Only the calculus teachers indicated that the half-course concepts were more difficult for the students to understand as compared to the concepts presented in the trigonometry portion of the Mathematics 31X course.

3. The majority of the teachers indicated that the concepts presented in each of the half-courses required more student thought than the concepts presented in trigonometry. Most of the teachers indicated that a background in matriculation mathematics was essential if the students were to benefit by studying the half-course. Indications were that this background was of more importance to those students studying calculus or matrix algebra as compared to students studying probability.

4. For each of the three experimental groups, the majority of teachers indicated that the half-courses provided good mathematical value for mathematically-inclined students. Most teachers especially those teaching the calculus and matrix algebra half-courses, reported that their students had demonstrated sufficent mathematical growth to justify the teaching of the half-course.

5. The teachers of the probability half-course stated that the prescribed text was a good choice for grade twelve students. Some calculus teachers indicated that the text did not provide enough exercises or sample problems and that the approach given in the text was not suitable for grade twelve students. Similar complaints were voiced by some of the teachers of matrix algebra.

6. It appears that a teacher of calculus or matrix algebra needs a stronger mathematical background than a teacher of probability in order to do an adequate job of teaching the half-course.

7. Only the teachers of matrix algebra indicated that their students showed a more favorable attitude toward matrix algebra than they showed toward trigonometry.

III. LIMITATIONS

There are a number of limitations that the reader must keep in mind when interpreting these results. First of all, the student sample used in this study cannot be considered as a random sample of the Mathematics 31 students in Alberta. This is also true of the teacher sample. However, different geographical areas of Alberta were represented.

Secondly, the instruments used in this study should be carefully inspected. Although a detailed explanation has been given of the construction of the questionnaires used, the reader is urged to study these instruments to ascertain if they are measuring the variables in question. Inspection of the final tests will also give the reader some indication of the rigor with which these half-courses were taught.[1]

At present, the Alberta high school mathematics curriculum is in a state of change. During the 1966-67 school year, an experimental course in Mathematics 30 was also conducted. Some of the topics in this new Mathematics 30 course

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[1]See Appendix J.

dealt with probability and statistics and many students studying the probability half-course received a double exposure to some of these topics.

Lastly, the students' final mark in the Mathematics 31X course was determined by using 75 per cent of the mark obtained on the January trigonometry final test and 25 per cent of the mark obtained on the half-course final given in June. Students in the regular Mathematics 31 course are passed or failed on the basis of a final test that they write in June. This grading procedure must be taken into account when comparisons involving attitude and anxiety are made between the control and experimental groups. However, this should not affect the comparative inferences drawn between any two experimental groups.

IV. CONCLUSIONS

On the basis of the results of the statistical analyses and in consideration of the teacher questionnaire results, these conclusions are drawn.

1. The matrix algebra half-course produced the most favorable student attitude toward mathematics.
2. Of those students in the experimental groups, the students who studied calculus showed the highest level of anxiety.

3. There was a significant positive relationship between achievement in trigonometry and achievement in each of the half-courses.
4. Within each of the half-courses, students of very high ability achieved significantly better than students of moderate ability. For the matrices group, the high ability students achieved significantly better than the moderate ability students.
5. University professors selected a knowledge of calculus as being most beneficial to freshman university students regardless of their major areas of interest. However, there were indications that these major areas of interest should be considered separately.
6. The participating teachers reacted favorably to the experiment and many indicated that they favored splitting the Mathematics 31 course into two parts. Only one teacher indicated a preference for returning to a full year of trigonometry. Teachers indicated that a half-course in calculus or matrix algebra is preferred to an half-course in probability as part of the Mathematics 31 course in probability as part of the Mathematics 31 course.
7. The extent of the relationship among the variables of attitude, anxiety, achievement, and ability varied

depending upon the experimental group in question. For each of the three experimental groups:

a) there was no significant relationship between ability and attitude towards the half-course.
b) there was a significant positive relationship between half-course achievement and ability.
c) the relationship between attitude and anxiety was negative but significant only for the calculus group.
d) the relationship between attitude and achievement was positive but significant for the calculus and probability groups only.
e) the relationship between anxiety and achievement was negative but significant for the calculus and probabil-groups only.
f) the relationship between anxiety and ability was negative but significant only for the calculus group.

V. RECOMMENDATIONS

The main purpose of this experiment was to determine the suitability of including three half-courses as part of the Mathematics 31 course. The following recommendations are based on the findings of this study.

The half-course in probability and statistics should not be included in the Mathematics 31 course. The main reason for this recommendation is that the new Mathematics 30 course (which was introduced throughout Alberta in September, 1967) would duplicate much of the material in the half-course. The effect of this overlap was demonstrated by the low, mean, attitude score recorded by the probability group. The responses of the probability teachers on the teacher questionnaire also

indicated that this duplication of material was undesirable.

The inclusion of a half-course in matrix algebra is recommended for the Mathematics 31 course. Of all the students participating in this study, those who studied matrix algebra showed the most favorable attitude. This result was supported by the teachers of matrix algebra who indicated that their students showed considerable interest in this material. One objective of new mathematics programs is to stimulate student interest in mathematics and to encourage more students to continue their study of mathematics. Exposing students to the nature of matrix algebra in high school would help meet this objective. Also, the results of the matrix algebra final test indicated that the participating students were capable of learning the concepts that were presented. Perhaps the half-course could be improved by reducing the amount of time spent on computational procedures thereby leaving more time for a study of the applications of matrix algebra.

The teaching of a half-course in calculus as part of the Mathematics 31 course is recommended. This study confirmed, that from the point of view of mathematical content, calculus has a wider application at the university level than either matrix algebra or probability. However, there were indications that the calculus course was the most dif-

ficult of the three that were offered. The calculus group exhibited a significantly greater mean anxiety level than either of the other two experimental groups and the teachers of calculus were the only ones who reported their half-course to contain more difficult concepts than those present in trigonometry. Some of the calculus teachers also reported that the textbook treatment of the subject tended to be too theoretical and too rigorous for grade twelve students. Despite the difficulty of the calculus material, many students showed satisfactory achievement on the final test. As with the matrices group, the extent of this achievement varied with the ability level of the student. The calculus half-course could also be improved by determining the degree of rigor with which the material should be taught and then choosing a suitable textbook.

Before a course such as Mathematics 31X is adopted provincially, serious consideration must be given to the students and teachers that would be involved. Many of the students involved in this study were selected on the basis of high ability and/or high mathematics achievement. In addition, the majority of teachers who participated in the Mathematics 31X program were well-qualified, mathematics teachers. Whether or not this calibre of student and teacher is available province-wide is doubtful. However, the Math-

ematics 31X course (as recommended above) should be continued in those schools where qualified personnel are available.

VI. IMPLICATIONS FOR FURTHER RESEARCH

One apparent implication for further research is the replication of an experiment similar to this one, especially where random sampling procedures could be employed for selecting students and teachers. It may also be advantageous to have a test, re-test situation for the attitude-anxiety questionnaire to measure changes in those variables associated with mathematics courses.

A follow-up study could also be conducted to determine what effect the experimental Mathematics 31X course has on student achievement in university mathematics courses. Many educators have expressed concern about offering non-credit, college-level courses at the high school level. The main objection raised is that the students will be required to repeat this material at college or university which may lead to boredom and eventually affect achievement. The present study offers an excellent starting point for a study of this nature.

Another question which should be investigated is the extent to which discovery teaching could be used in teaching

courses similar to the half-course mentioned in this study. Most research dealing with discovery teaching has been restricted to the elementary or junior high school grades.

The need for studies involving the evaluation of new mathematics programs is immense. Although many researchers tend to avoid this controversial area, the investigator believes that the value of this type of research cannot be overestimated. Effective curriculum planning requires time, patience, and a wealth of information which research can supply.

BIBLIOGRAPHY

A. BOOKS

Campbell, William Giles. Form and Style in Thesis Writing. Boston: Houghton Mifflin Company, 1954.

Drummond, M. Psychology and Teaching of Number. New York: World Book Company, 1922.

Davis, Philip J. The Mathematics of Matrices. Waltham, Massachusetts: Blaisdell Publishing Company, 1965.

Ebel, Robert L. Measuring Educational Achievement. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965.

Eysenck, H. J. The Dynamics of Anxiety and Hysteria. London: Routledge and Keegan Paul, 1957.

Fehr, Howard F., Lucas Bunt, and George Grossman. An Introduction into Sets, Probability, and Hypothesis Testing. Boston: D. C. Heath and Company, 1965.

Ferguson, George A. Statistical Analysis in Psychology and Education. New York: McGraw-Hill Book Company, Inc., 1966.

Fraser, Dorothy M. Current Curriculum Studies in Academic Subjects. Washington: National Education Association, 1962.

Freud, Sigmund. Inhibitions, Symptoms and Anxiety. London: The Hogarth Press, 1948.

Guilford, J. P. Fundamental Statistics in Psychology and Education. New York: McGraw-Hill Company, Inc., 1965.

Heath, Robert W. (ed.). New Curricula. New York Harper and Row, Inc., 1964.

Kenney, J. F. and E. S. Keeping. Mathematics of Statistics, Part One. Princeton: A. Van Nostrand Company, Inc., 1954.

Krathwohl, David R., Benjamin Bloom, and Bertram Masiu. Taxonomy of Educational Objectives, The Classification of Educational Goals, Handbook II: Affective Domain. New York: David McKay Company, Inc., 1956.

Lang, Serge. A First Course in Calculus. Reading Massachusetts: Addison-Wesley Publishing Company, Inc., 1965.

Mace, C. A. The Psychology of Study. London: Meuthen and Co., Ltd., 1932.

Mowrer, O. H. Learning Theory and Personality Dynamics. New York: Ronald Press, 1950.

Remmers, H. H., N. L. Gage, and J. Francis Rummel. A Practical Introduction to Measurement and Evaluation. New York: Harper and Row, 1965.

Saylor, John G. and William M. Alexander. Curriculum Planing for Modern Schools. New York: Holt, Rinehart, and Winston, 1966.

Sellitz, Claire et al. Research Methods in Social Relations. United States: Holt, Rinehart and Winston, 1963.

Taba, Hilda. Curriculum Development: Theory and Practice. Harcourt, Brace and World, 1962.

Winer, B. J. Statistical Principles in Experimental Design. New York: McGraw-Hill Company, Inc., 1962.

B. NEWSLETTERS, REPORTS, ADDRESSES, PUBLICATIONS, AND YEARBOOKS

Alpert, R., G. Stellwagon, and D. Becker. "Psychological Factors in Mathematics Education," Newsletter No. 15, pp. 17-24. Stanford, California: Leland Stanford University, 1963.

Biggs, J. B. Anxiety, Motivation, and Primary School Mathematics, Occasional Publication No. 7, The Information Service of the National Foundation for Educational Research in England and Wales, 1962.

Buros, Oscar, K. (ed.). The Fifth Mental Measurements Yearbook. Highland Park, N. J.: The Gryphon Press, 1959

Commission on Mathematics. A Summary of the Report of the Commission on Mathematics. Princeton: College Entrance Examination Board, 1959.

Corcoran, Mary and E. Glenadine Gibb. "Appraising Attitudes In the Learning of Mathematics," Evaluation in Mathematics, Twenty-sixth Yearbook, pp. 105-121. Washington: The National Council of Teachers of Mathematics, 1961.

Cronbach, Lee J. "Evaluation for Course Improvement," New Curricula, pp. 231-248. R. W. Heath (ed.), New York: Harper and Tow, Inc., 1964.

Educational Services Incorporated. Goals for School Mathematics. The Report of the Cambridge Conference on School Mathematics. Boston: Houghton Mifflin Company, 1963.

Fehr, Howard F. "Teaching High School Mathematics," What Research Says to the Teacher, No. 9. National Education Association of the United States, 1955.

Goodlad, John I. "Direction and Redirection for Curriculum Change," Curriculum Change: Direction and Process, pp. 4-12. R. R. Leeper (ed.) Washington: Association for Supervision and Curriculum Development, 1966.

Lighthall, Frederick E. "Anxiety as Related to Thinking and Forgetting," What Research Says to the Teacher, No. 30. National Education Association of the United States, May, 1964.

National Council of Teachers of Mathematics, An Analysis of New Mathematics Programs, 1964.

Woodby, Lauren G. Emerging Twelfth-Grade Mathematics Programs. U. S. Department of Health, Education, and Welfare. Washington: Government Printing Office, 1965.

C. PERIODICALS

Aiken, Lewis R., and Ralph M. Dreger, "The Effects of Attitude on Performance in Mathematics," Journal of Educational Psychology, 52: 19-24, February, 1961.

Allendoerfer, C. B. "The Case Against Calculus," The Mathematics Teacher, 56: 482-485, November, 1963.

Barakat, M. K. "Factors Underlying the Mathematical Abilities of Grammar School Pupils," British Journal of Educational Psychology, 21:239-240, November, 1951.

Beninati, A. "It's Time to Take a Closer Look at High School Calculus," The Mathematics Teacher, 59:29-39, January, 1966.

Biggs, J. B. "Attitudes to Arithmetic--Number Anxiety," Educational Research I, 3:6-21, June, 1959.

Blank, Albert A. "Remarks on the Teaching of Calculus in the Secondary School," The Mathematics Teacher, 53:537-539, November, 1960.

Bridges, Charles M. Jr. "Application of Elementary Statistics in Analysis of Data by Selected Secondary School Students," Dissertation Abstracts, 20:1223.

Brown, Kenneth D., and Theodore L. Abell. "Research in the Teaching of High School Mathematics," The Mathematics Teacher, 59:53-57, January, 1966.

Brown, Robert S. "Survey of Ohio College Opinions With Reference to High School Mathematics Programs," The Mathematics Teacher, 56:245-247, April, 1963.

Buchanan, O. L. Jr. "Opinions of College Teachers of Mathematics Regarding Content of the Twelfth-Year Course in Mathematics," The Mathematics Teacher, 58:223-225, March, 1965.

Chastain, Eleanor, "Objectives of Experimental Courses in Elementary Mathematics," School Science and Mathematics, 65:49-55, January, 1965.

De Michele, John H. "The Interpretation of Anxiety by Various Psychotherapeutic Schools," Journal of Consulting Psychology, 18:47-52, No. 1, 1954.

Dreger, Ralph M. and L. R. Aiken, "The Identification of Number Anxiety in a College Population," Journal of Educational Psychology, 48:344-351, 1957.

Dutton, W. H. "Attitudes of Junior High School Pupils," School Review, 64:18-22, January, 1956.

Gibbons, K. C. and R. D. Savage, "Intelligence, Study Habits, and Personality Factors in Academic Success--A Preliminary Report," The Durham Research Review, 5:8-12, September, 1965.

Grossman, G. "Probability and Statistics in the Twelfth-Year?" The Mathematics Teacher, 54:540-546, November, 1961.

Hungerman, Ann D. "Achievement and Attitude of Sixth Grade Pupils in Conventional and Contemporary Mathematics Programs," The Arithmetic Teacher, 14:30-39, January, 1967.

Johnson, Donovan A. "Attitudes in Mathematics Classrooms," School Science and Mathematics, 57:113-120, February, 1957.

Johnson, Donovan A. "Evaluating a School Mathematics Curriculum," School and Society, 90:424-426, December, 1962.

Johnson, Sonia A. "Some Selected Classroom Variables and Their Relationship to Mathematics Achievement in Central Minnesota and the Greater London Area," Dissertation Abstracts, 27:139-A, No. 1.

Leissa, Arthur W., and Robert C. Fisher, "A Summary of Teachers' Opinions of a Revisied Mathematics Curriculum," The Mathematics Teacher, 53:113-118, February, 1960.

Lovell, K., and G. White, "Some Influences Affecting Choice of Subjects in School and Training College," British Journal of Educational Psychology, 28:15-24, February, 1958.

Lynn, R., and T, E, Gordon. "The Relation of Neuroticism and Extroversion to Intelligence and Educational Attainment," British Journal of Educational Psychology, 3: 194-203, June, 1961.

McCandless, B., and A. Castaneda. "Anxiety in Children, School Achievement, and Intelligence," Child Development, 27:379-382, September, 1956.

McKillip, William D. "The Effects of High School Calculus on Students' First Semester Calculus Grades at the University of Virginia," The Mathematics Teacher, 59:470-472, May, 1961.

National Council of Teachers of Mathematics, Secondary School Curriculum Committee. "The Secondary Mathematics Curriculum," The Mathematics Teacher, 53:389-417, May 1959.

Olander, Clarence. "Let's Teach Statistics," The Mathematics Teacher, 51:253-260, April, 1958.

Philips, B. N., E. Hindsman, and E. Jennings. "Influences Of Intelligence on Anxiety and Perception of Self and Others," Child Development, 31:41-46, March, 1960.

Poffenberger, T., and D. Norton. "Factors Determining Attitudes Towards Mathematics and Arithmetic," The Arithmetic Teacher, 3:113-116, April, 1956.

Poffenberger, T., and D. Norton. "Factors in the Formation of Attitudes Towards Mathematics," The Journal of Educational Research, 52:171-177, January, 1959.

Pritchard, R. "The Relative Popularity of Secondary School Subjects at Various Ages," British Journal of Educational Psychology, 5:157-179, June, 1935.

Reese, H. W. "Manifest Anxiety, Achievement, and Test Performance," Journal of Educational Psychology, 52: 132-135, 1961.

Sarason, S. B., et al. "A Test Anxiety Scale for Children," Child Development, 29:105-113, March, 1958.

Spielberger, C. D., and W. Katzenmeyer. "Manifest Anxiety, Intelligence, and College Grades," Journal of Consulting Psychology, 23:278, No. 3, 1959.

Stakenas, R. G. "Evaluative Stress, Fear of Failure, and Academic Achievement," Dissertation Abstracts, 27:601-A, No. 2.

Williams, H. E. "The Place of Probability and Statistics in the Mathematics Curriculum of the Senior High School and Junior College," The Mathematics Teacher, 54:316-320, May, 1961.

Wrigley, J. "The Factorial Nature of Ability in Elementary Mathematics," British Journal of Educational Psychology, 28:61-78, February, 1958.

D. PUBLISHED TESTS AND TEST MANUALS

Cooperative Test Division of the Educational Testing Service. Cooperative School and College Ability Tests, Technical Report. Princeton, N. J.: Educational Testing Service, 1957.

Educational Testing Service. Cooperative School and College Ability Tests, School Ability Test, Level 1B. Princeton: Cooperative Test Division, Educational Testing Service, 1955.

E. UNPUBLISHED MATERIALS

Coon, Lewis. "SMSG Mathematics as a Factor Influencing Success in Freshman Calculus," Unpublished Doctoral dissertation, Oklahoma State University, Stillwater, 1963.

Ellingson, J. B. "Evaluation of Attitudes of High School Students Towards Mathematics." Unpublished Doctoral dissertation, University of Oregon, Eugene, 1962.

Fredstrom, Paul N. "An Evaluation of the Accelerated Mathematics Program in the Lincoln, Nebraska Public Schools." Unpublished Doctoral dissertation, University of Nebraska Teachers College, Lincoln, 1964.

Freeman, W. A. "An Enquiry into the Attitudes of Secondary School Boys and Girls Towards Arithmetic." Unpublished Master's thesis, University of Birmingham, Birmingham, 1948.

Harrison, D. B. "An Analysis of the Effectiveness of Three Mathematics Programs at the Grade Eight Level." Unpublished Master's thesis, University of Alberta, Edmonton, 1964.

Lang, Robert W. "A Study of an Accelerated Mathematics Program," Unpublished Doctoral dissertation, Wayne State University, Detroit, 1962.

Remai, H. A. "An Experimental Investigation Comparing Attitudes Toward Mathematics of Modern and Traditional Mathematics Students at the Junior High School Level." Unpublished Master's thesis, University of Alberta, Edmonton, 1965.

Wick, Marshall E. "A Study of the Factors Associated With Achievement in First-Year College Mathematics." Unpublished Doctoral dissertation, University of Minnesota, Minneapolis, 1963.

APPENDIX A

REMAI'S ATTITUDE SCALE

The best answer to each statement is your own first impression. There are no right or wrong answers. Think carefully, but do not spend too much time on any one question. Let your own personal experience guide you to choose the answer you feel about each statement.

R_{pbi} Please mark a response for every statement.

.36 1. I find most mathematics lessons:

a) extremely interesting.
b) quite interesting.
c) interesting.
d) not very interesting.
e) not interesting at all.

.38 2. A knowledge of mathematics for any job at all is:

a) most important.
b) very important.
c) quite important.
d) of small importance.
e) not important.

.55 3. If I did not have to take mathematics, I would like school:

a) much less.
b) a little less.
c) same as now.
d) a little better.
e) much better.

.31 4. Mathematics is:

a) the most important subject.
b) one of the more important subjects.
c) just as important as any other subject.
d) not as important as some of the other subjects.
e) the least important subject.

.51 5. I find problem solving:

a) extremely interesting.
b) quite interesting.
c) interesting.
d) not very interesting.
e) not interesting at all.

.26 6. When I have difficulty with a new topic in my mathematics course, I ask my teacher to clarify the section:

a) very frequently.
b) frequently.
c) sometimes.
d) hardly ever.
e) never.

.44 7. If books about mathematics were available, I would:

a) read most of them.
b) read some of them.
c) look at the diagrams and pictures.
d) page through some of them.
e) never look at them.

.45 8. If someone says mathematics classes are worthless and a waste of time, I would:

a) strongly disagree.
b) tend to disagree.
c) not take a side.
d) tend to agree.
e) strongly agree.

.53 9. When I do my homework, my mathematics is:

a) always done first.
b) often done first.
c) usually done first.
d) sometimes done first.
e) never done first.

.58 10. I find mathematical puzzles:

a) extremely interesting.
b) quite interesting.
c) sometimes interesting.
d) not very interesting.
e) not interesting at all.

.59 11. I would be interested in taking other subjects that make use of:

a) a great deal of mathematics
b) quite a bit of mathematics.
c) some mathematics.
d) a little mathematics
e) no mathematics.

.29 13. If given the opportunity to join one of the following clubs, I would prefer a:

a) mathematics club.
b) science club (physics).
c) science club (chemistry).
d) science club (geology).
e) literary club.

.35 14. If I could receive one the following magazines for a year, I would pick:

a) a mathematics magazine for high school students.
b) a magazine combining science and mathematics for high school students.
c) a science magazine for high school students.
d) a geology magazine for high school students.
e) a literary magazine for high school students.

.36 15. When I study my mathematics course, I most often:

a) make written summaries of the sections covered.
b) do additional problem solving.
c) do many drill questions.
d) memorize the formulas given the text.
e) look over some work done previously.

.43 16. If I listed by courses in order of preference, I would place mathematics:

a) first.
b) second.
c) third.
d) fourth.
e) fifth.

.39 17. Whenever mathematical problems are presented to us for solving, I get:

a) a great deal of satisfaction in working them out.
b) quite a bit of satisfaction in working them out.
c) some satisfaction in working them out.
d) very little satisfaction in working them out.
e) no satisfaction in working them out.

.28 18. My mathematics course has made:

a) mathematics enjoyable for me.
b) mathematics a pleasant course.
c) me feel indifferent towards mathematics.
d) mathematics classes an unccmfortable experience for me.
e) me strongly dislike mathematics.

.24 19. I feel my mathematics teacher:

a) enjoys teaching mathematics.
b) gets some pleasure in teaching mathematics.
c) gets some satisfaction in teaching mathematics.
d) neither likes or dislikes teaching mathematics.
e) dislikes teaching mathematics.

.51 20. When I do my mathematics homework, I am usually:

a) extremely interested.
b) interested.
c) somewhat interested.
d) not too interested.
e) not interested at all.

.45 21. When we start a new topic in mathematics, I am usually:

a) keenly interested.

b) interested.
c) somewhat interested.
d) not too interested.
e) not interested at all.

.31 22. The average amount of time I spend on homework assignments in mathematics takes the following time per day:

a) more than one hour.
b) 3/4 hour to one hour.
c) 1/2 hour to 3/4 hour.
d) 1/4 hour to 1/2 hour.
e) 0 hours to 1/4 hour.

.26 23. When I get an assignment in mathematics:

a) I do it immediately.
b) I do it eventually.
c) I may get it done.
d) I put it off as long as possible.
e) I don't do it.

.36 24. Most of my work in this class is done:

a) to satisfy my curiosity about mathematics.
b) to gain competence in mathematics.
c) to get a good mark.
d) to just pass the class.
e) to put in the time alloted to mathematics.

.23 25. During mathematics lessons, I feel:

a) extremely confident in myself.
b) quite confident in myself.
c) confident in myself.
d) a little unsure of myself.
e) very unsure of myself.

R_{pbi} represents the point biserial correlation coefficients of the items.

Remai also reported the following coefficients for his attitude scale:

1. Between teacher ratings and attitude scores .43
2. Between attitude scores and achievement in mathematics .37
3. Test-re-test .77
4. Internal consistency coefficient, based on analysis of variance .86

APPENDIX B

PILOT STUDY ATTITUDE ITEMS

1. The main reason for my taking the Mathematics 31 course is:

    a) to learn more mathematics.
    b) to gain competence in mathematics.
    c) to get a good mark.
    d) for university entrance.
    e) to obtain five credits.

2. I find myself bored with the Mathematics 31 course:

    a) very seldom.
    b) seldom.
    c) occasionally.
    d) often.
    e) very often.

3. If I listed by Grade Twelve courses in order of preference, I would place Mathematics 31:

    a) first.
    b) second.
    c) third.
    d) fourth.
    e) fifth.

4. If I had to study another course similar to the Mathematics 31 course, I would be:

    a) very happy.
    b) happy.
    c) neither happy or disappointed.
    d) disappointed.
    e) very disappointed.

5. I find that solving Mathematics 31 problems is:

    a) extremely interesting.
    b) very interesting.
    c) somewhat interesting.
    d) not very interesting.
    e) not interesting at all.

6. If I were not taking Mathematics 31, I would like school:

    a) much less.

b) a little less.
c) the same as now.
d) a little better.
e) much better.

7. When we start a new topic in Mathematics 31, I am usually:

a) keenly interested.
b) interested.
c) somewhat interested.
d) not too interested.
e) not interested at all.

8. If more books about the Mathematics 31 course were available, I would:

a) read most of them.
b) read some of them.
c) look at the diagrams and pictures.
d) page through some of them.
e) never look at them.

9. Some people say that the Mathematics 31 course is worthless and a waste of time. I would:

a) strongly disagree.
b) tend to disagree.
c) not take a side.
d) tend to agree.
e) strongly agree.

10. If I continue my formal education at another institution (university, technical school, etc.), I hope to take courses that make use of:

a) a great deal of Mathematics 31.
b) quite a bit of Mathematics.31.
c) some Mathematics 31.
d) very little of Mathematics 31.
e) no Mathematics 31.

11. The amount of time I spend on Mathematics 31 homework assignments daily is:

a) more than 1 hour.
b) 3/4 of an hour to 1 hour.
c) 1/2 of an hour to 3/4 of an hour.
d) 1/4 of an hour to 1/2 of an hour.
e) 0 hours to 1/4 of an hour.

12. If I accidently come upon a mathematical puzzle or game in a book, magazine, or newspaper, I:

    a) always try to solve the puzzle or game in a book
    b) frequently try to solve the puzzle or game.
    c) occasionally try to solve the puzzle or game.
    d) rarely try to solve the puzzle or game.
    e) never try to solve the puzzle or game.

13. While I am doing my Mathematics 31 homework, I usually find myself:

    a) keenly interested.
    b) interested.
    c) somewhat interested.
    d) not too interested.
    e) not interested at all.

14. Compared to the topics that I am studying in Mathematics 30, I think the Mathematics 31 course is:

    a) much more interesting.
    b) more interesting.
    c) equally as interesting.
    d) not as interesting.
    e) not nearly as interesting.

15. I find the Mathematics 31 course:

    a) very interesting.
    b) interesting.
    c) no more interesting than any other course.
    d) not very interesting.
    e) not interesting at all.

16. Many people claim that a knowledge of Mathematics 31 is helpful in most occupations. I would:

    a) definitely agree.
    b) tend to agree.
    c) not take a side.
    d) tend to disagree.
    e) definitely disagree.

17. When I do my homework at night, I prefer to do my Mathematics 31 homework:

    a) first.
    b) second.
    c) third.
    d) fourth.
    e) fifth.

18. Because I am taking Mathematics 31, I feel that my interest in mathematics has:

    a) increased greatly.
    b) increased somewhat.
    c) remained the same.
    d) decreased somewhat.
    e) decreased greatly.

PILOT STUDY ANXIETY ITEMS

1. During most of the Mathematics 31 classes, I usually feel:

    a) very relaxed.
    b) relaxed.
    c) a bit tense.
    d) quite tense.
    e) very tense.

2. While I am doing my Mathematics 31 homework, I am usually able to concentrate:

    a) very well.
    b) well.
    c) to some extent.
    d) not too well
    e) very poorly.

3. Compared to other tests that I write, I feel that Mathematics 31 tests upset me:

    a) very little.
    b) a little.
    c) about the same as the others.

d) a little more than the others.
e) much more than the others.

4. If we are studying a difficult section of the Mathematics 31 course and I am having trouble understanding the material, I find that this bothers me:

a) very seldom.
b) seldom.
c) sometimes.
d) often.
e) very often

5. I have a tendency to "freeze-up" when I see a Mathematics 31 problem:

a) very rarely.
b) rarely.
c) sometimes.
d) often.
e) very often.

6. While I am writing a Mathematics 31 test, I usually feel that I am doing:

a) very well.
b) well.
c) average.
d) not very well.
e) very poorly.

7. After I have written a Mathematics 31 test, I like to talk over the test with other students:

a) always.
b) very often.
c) sometimes.
d) very seldom.
e) never.

8. I find myself thinking about failing Mathematics 31:

a) very rarely.
b) rarely.
c) sometimes.
d) often.
e) very often.

9. If my parents or other adults ask me about the Mathematics 31 course, I tell them:

   a) very willingly.
   b) willingly.
   c) casually.
   d) hesitantly.
   e) very hesitantly.

10. When my teachers asks me to explain a problem to the class or to write it on the blackboard, I feel:

    a) very confident
    b) confident.
    c) indifferent
    d) uneasy.
    e) very uneasy.

11. If my teacher stops at my desk to observe me while I am doing a problem, I find that this upsets me:

    a) very rarely.
    b) rarely.
    c) sometimes.
    d) often
    e) very often.

12. When I am doing Mathematics 31 problems, I have a tendency to become flustered or upset:

    a) very seldom.
    b) seldom.
    c) occasionally.
    d) often.
    e) very often.

APPENDIX C

CORRELATION MATRIX FOR ATTITUDE ITEMS

Item No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																				
2	26																			
3	11	45																		
4	29	59	57																	
5	28	46	37	41																
6	04	38	63	48	45															
7	19	41	22	50	37	17														
8	14	16	23	23	38	15	46													
9	30	31	25	53	29	27	22	23												
10	-02	23	53	45	27	55	20	21	35											
11	-04	14	00	07	-03	-22	46	24	-19	-14										
12	18	10	11	15	13	09	-14	-09	16	35	-06									
13	18	63	50	52	48	42	25	10	24	30	03	24								
14	13	23	45	33	21	42	11	22	25	40	-06	04	42							
15	11	54	60	54	46	46	35	26	29	53	02	09	11	60						
16	04	17	20	30	13	28	25	29	42	32	02	03	17	39	17					
17	19	04	06	14	10	08	27	11	20	12	12	03	31	39	34	20				
18	-05	43	48	52	35	54	30	22	38	38	-12	-03	48	41	55	30	20			
19	36	65	68	76	60	60	55	48	55	61	14	25	70	62	77	48	42	62		
20	-13	41	34	39	24	31	29	23	21	43	14	23	52	33	50	50	27	40	57	

Note: Decimal point has been omitted.
Item 19 corresponds to total test score.
Item 20 corresponds to teacher's rating.

CORRELATION MATRIX FOR ANXIETY ITEMS

Item No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1														
2	27													
3	21	17												
4	06	-04	05											
5	10	26	21	29										
6	29	42	47	09	22									
7	30	19	-22	05	23	19								
8	31	33	12	24	41	33	25							
9	14	29	22	05	03	31	14	34						
10	55	40	23	-05	17	31	26	48	41					
11	17	28	-04	14	26	20	03	21	12	24				
12	22	22	25	22	51	40	04	56	17	31	22			
13	56	59	42	30	56	66	38	72	50	68	44	63		
14	-09	-24	04	14	-09	-12	-11	-07	04	-18	-18	-16	-16	

Note: Decimal points are omitted
Item 13 represents total test score
Item 14 represents teacher rating

APPENDIX D

TEACHER'S GUIDELINE

Your evaluation of the students' attitude towards Mathematics 31 is needed for the validation of the attitude scale. To help make your observations more objective, a guideline has been prepared. On the basis of these observations, opinions that the students may have expressed, and any other knowledge you may have, please rate each student on a five point scale with 5 indicating the most favorable attitude and 1 the least favorable.

1. Contributes to group discussion by asking thought-provoking questions or supplying relevant information and ideas.

2. Finds pleasure in doing mathematical puzzles and in playing real number games.

3. Shows a desire and interest in learning Mathematics 31.
---attentive in class.
---homework and classwork completed neatly and carefully.
---doesn't wait until the last minute to complete assignments

4. Willingness to spend time, energy, attention to build insight, creativity, discovery beyond the requirements of the course.
---suggests and works out alternative methods for problems.
---does extra reading on his own.
---solves optional problems and brings mathematical problems to school.

5. Will study Mathematics 31 because he enjoys it, he gets satisfaction from knowing mathematical ideas, he feels rewarded when he attains mathematical competence.
---takes pride in his work.
---doesn't give up quickly.
---appreciates a logical proof.

6. Curiosity about the unique structure of Mathematics 31 and a continuing desire to understand the structure of mathematics.
---wants to know "why" and not just "how"
---strives for precision and a logical order in his proof.
---discusses mathematical ideas and concepts with his friends.

APPENDIX E

TEACHER'S GUIDELINE

Your evaluation of the student's anxiety associated with the Mathematics 31 course is needed for the validation of the anxiety scale. To help make your observations more objective a guideline has been prepared. On the basis of this guideline, your personal contact with the students, and any other knowledge you may have, please rate each student on a five point scale with 5 indicating the highest anxiety level and 1 indicating the lowest anxiety level.

1. Does the student exhibit unwarranted fidgeting (squirming, restless behavior) when called upon to answer questions in class?

2. Does the student work better in a situation in which he can take his time than in one in which there is time pressure?

3. Does the student seem to be tense or insecure during class periods?

4. Does the student become noticeably flustered during Mathematics 31 examinations?

5. Is the student's work usually done in an orderly manner or does the student frequently cross out work and start over? (i.e. Is the student's work noticeably disorganized?)

6. Does the student have difficulty in concentrating for any length of time?

7. Has the student expressed concern over his progress in the Mathematics 31 course to the extent where he has become upset or worried about it? Do you feel that the student is worrying to the point that it is interfering with his progress in the course?

8. Does the student show perserverance when doing assignments or studying difficult sections of the course? (i.e. The student does not tend to "get away from it all")

9. Does the student's pattern of thinking tend to be very rigid or inflexible?

10. Does the student appear dejected or depressed during Mathematics 31 classes.

APPENDIX F

FINAL ATTITUDE-ANXIETY QUESTIONNAIRE

1. I find myself bored with the Mathematics 31 course:

   a) very seldom.
   b) seldom.
   c) occasionally.
   d) often.
   e) very often.

2. If I listed my grade twelve courses in order of preference, I would place Mathematics 31:

   a) first
   b) second.
   c) third.
   d) fourth.
   e) fifth.

3. If I had to study another course similar to the Mathematics 31 course, I would be:

   a) very happy.
   b) happy.
   c) neither happy nor disappointed.
   d) disappointed
   e) very disappointed.

4. I find that solving Mathematics 31 problems is:

   a) extremely interesting.
   b) very interesting.
   c) somewhat interesting.
   d) not very interesting.
   e) not interesting at all.

5. If I were not taking Mathematics 31, I would like school:

   a) much less.
   b) a little less.
   c) the same as now.
   d) a little better
   e) much better.

6. When we start a new topic in Mathematics 31, I am usually:

   a) keenly interested.
   b) interested.
   c) somewhat interested.
   d) not too interested.
   e) not interested at all.

7. If more books about the Mathematics 31 course were available, I would:

   a) read most of them.
   b) read some of them.
   c) look at the diagrams and pictures.
   d) page through some of them.
   e) never look at them.

8. Some people say that the Mathematics 31 course is worthless and a waste of time. I would:

   a) strongly disagree.
   b) tend to disagree.
   c) not take a side.
   d) tend to agree.
   e) strongly agree.

9. If I continue my formal education at another institution (university, technical school, etc.), I hope to take courses that make use of:

   a) a great deal of Mathematics 31.
   b) quite a bit of Mathematics 31.
   c) some Mathematics 31.
   d) very little Mathematics 31.
   e) no Mathematics 31.

10. While I am doing my Mathematics 31 homework, I ususally find myself:

    a) keenly interested.
    b) interested.
    c) somewhat interested.
    d) not too interested.
    e) not interested at all.

11. Compared to the topics that I am studying in Mathematics 30, I think the Mathematics 31 course is:

a) much more interesting.
b) more interesting.
c) equally as interesting.
d) not as interesting.
e) not nearly as interesting.

12. I find the Mathematics 31 course:

a) very interesting.
b) interesting.
c) no more interesting than any other course.
d) not very interesting.
e) not interesting at all.

13. Many people claim that a knowledge of Mathematics 31 is helpful in most occupations. I would:

a) definitely agree.
b) tend to agree.
c) not take a side.
d) tend to disagree.
e) definitely disagree.

14. When I do my homework at nignt, I <u>prefer</u> to do my Mathematics 31 homework:

a) first.
b) second.
c) third.
d) fourth.
e) fifth.

15. Because I am taking Mathematics 31, I feel that my interest in mathematics has:

a) increased greatly.
b) increased somewhat.
c) remained the same.
d) decreased somewhat.
e) decreased greatly.

16. During most of the Mathematics 31 classes, I usually feel:

    a) very tense.
    b) quite tense.
    c) a bit tense.
    d) relaxed.
    e) very relaxed.

17. While I am doing my Mathematics 31 homework, I am usually able to concentrate:

    a) very poorly.
    b) not too well.
    c) to some extent.
    d) well.
    e) very well.

18. Compared to other tests that I write, I feel that Mathematics 31 tests upset me:

    a) much more than the others.
    b) a little more than the others.
    c) about the same as the others.
    d) a lttle less than the others.
    e) much less than the others.

19. If we are studying a difficult section of the Mathematics 31 course and I am having trouble understanding the material, I find that this bothers me:

    a) very often.
    b) often.
    c) sometimes.
    d) seldom.
    e) very seldom.

20. I have a tendency to "freeze-up" when I see a Mathematics 31 problem:

    a) very often.
    b) often.
    c) sometimes.
    d) rarely.
    e) very rarely.

21. While I am writing a Mathematics 31 test, I usually feel that I am doing:

    a) very poorly
    b) not very well.
    c) average.
    d) well.
    e) very well.

22. After I have written a Mathematics 31 test, I like to talk over the test with other students:

    a) never.
    b) very seldom.
    c) sometimes.
    d) very often.
    e) always.

23. I find myself thinking about failing Mathematics 31:

    a) very often.
    b) often.
    c) sometimes.
    d) very often.
    e) always.

24. If my parents or other adults ask me about the Mathematics 31 course, I tell them:

    a) very hesitantly
    b) hesitantly,
    c) casually.
    d) willingly.
    e) very willingly.

25. When my teacher asks me to explain a Mathematics 31 problem to the class or to write it on the blackboard, I feel:

    a) very uneasy.
    b) uneasy.
    c) indifferent.
    d) confident.
    e) very confident

26. If my teacher stops at my desk to observe me while I am doing a Mathematics 31 problem. I find that this upsets me:

    a) very often.
    b) often.
    c) sometimes.
    d) rarely
    e) very rarely.

27. When I am doing Mathematics 31 problems, I have a tendency to become flustered or upset:

    a) very often.
    b) often.
    c) occasionally.
    d) seldom.
    e) very seldom.

APPENDIX G

11121-82 Avenue,
Edmonton, Alberta,
March 14, 1967.

Dear Sir or Madam:

The purpose of this letter is to ask for your co-operation in completing a brief questionnaire. As a graduate student majoring in Secondary Education, I have chosen for my thesis topic, "An Investigation of Three Mathematics Half-Courses at the Grade Twelve Level." As part of this investigation, I would like to determine the reaction of professional educators to these three experimental courses. Before proceeding further, I will briefly outline these three courses.

During the present school year, under the guidance of the Senior High School Mathematics Subcommittee, three experimental half-courses are being taught in various Alberta High Schools at the Grade Twelve level. These courses are: (1) Introductory Calculus, (2) An Introduction to Sets, Probability, and Hypothesis Testing, and (3) Introductory Linear Algebra. Approximately one hundred students are involved in each of these courses and each course is being offered for a five month period. During the first five months of the school year, the students enrolled in these experimental courses studied plane trigonometry.

One need for experimenting with new material at the high school level arises from the changing Junior High School mathematics curriculum. As a result of these changes, junior high school students are now studying many topics that were once reserved for the high school student. This allows for new material to be introduced into the high school and particularly at the Grade Twelve level. The natural question that follows is "What topics more most appropriate at this level?"

This leads us back to the questionnaire and why you have received it. Firstly, nearly all of the students involved in the experimental courses intend to enter university. Secondly, there has never been a time when the impact of mathematics on so many fields--science, technology, humanities, research,--has been so great. Certainly one objective of the high school mathematics program is to prepare students for further learning at academic institutions such as universities. The high school, therefore, has an obligation to teach the material that will be most beneficial to students entering university. The purpose of this questionnaire is to determine which of the three experimental courses, as judged by university professors, provides the best mathematical background for university freshmen. This

questionnaire is being circulated to ninety-eight professors at the University of Alberta who are instructing in different faculties or departments. You have been randomly selected as one of these professors.

Attached to the questionnaire you will find a detailed outline of the content for each experimental course. This outline should provide some assistance as you complete the questionnaire. Many of you may feel that mathematics is completely unrelated to your field of study. While this may be true, I hope that you will still complete the questionnaire since this information is also valuable to curriculum planners. The completed questionnaire may be returned by using the self-addressed envelope that is provided.

I would like to thank you for your co-operation and time in reading this material. While the material is still fresh in your mind, why not complete the questionnaire and mail it? I would appreciate it very much if you would return the completed questionnaire before the first of April.

Yours truly,

Daniel J. Tessari

COURSE OUTLINES

A. Calculus:
Text: Lang, Serge. A First Course in Calculus. Addison-Wesley Publishing Company, Inc., 1965.

Content

Chapter 1 Numbers and Functions
1. Integers, rational numbers, and real numbers
2. Inequalities
3. Functions
4. Powers

Chapter 2 Graphs and Curves
1. Coordinates
2. Graphs
3. The straight line
4. Distance between two points
5. Curves and equations
6. The circle
7. The parabola. Change of coordinates
8. The hyperbola

Chapter 3 The Derivative
1. The slope of a curve
2. The derivative
3. Limits
4. Powers
5. Sums, products, and quotients
6. The chain rule
7. Rate of change

Chapter 4 Sine and Cosine
1. The sine and cosine function
2. The graphs
3. Addition formula
4. The derivatives
5. Two basic limits

Chapter 5 The Mean Value Theorem
1. The maximum and minimum theorem
2. Existence of maxima and minima
3. The mean value theorem
4. Increasing and decreasing functions

Chapter 6 Sketching Curves
1. Behaviour as x becomes very large
2. Curve sketching
3. Polar coordinates
4. Parametric curves

Chapter 7 Inverse Functions
1. Definition of inverse functions
2. Derivative of inverse functions
3. The arcsine
4. The arctangent

Chapter 8 Exponents and Logarithms
1. The logarithm
2. The exponential function
3. The general exponential function
4. Order of magnitude
5. Some applications

B. Sets, Probability, and Hypothesis Testing.

Text: Fehr, Howard F., Lucas N. H. Bunt, and George Grossman. An Introduction into Sets, Probability, and Hypothesis Testing, Boston: D. C. Heath and Company, 1965.

Content

Chapter 1 Sets and Operations
1. Sets--Set Designation
2. Ordered Pairs, Equal Sets, Subsets, The Null Set
3. More on Subsets
4. Operations: Binary and Unary
5. Binary Connectives: "Or" and "And"
6. Binary Operations on Sets--Intersection, Union
7. Unary Operations on Sets--Complementation
8. Venn Diagrams
9. Properties of Union and Intersection
10. Other Properties of Set Operations
11. Mapping of Sets, Types of Mappings
12. Binary Relations, Equivalence Relations
13. Equivalent Sets, Cardinal Number of a Set
14. Set Function
15. Additive Set Functions and Their Properties
16. Ordered Pairs and Product Sets
17. Fundamental Principle of Counting.

<u>Chapter 2 Permutation and Combinations</u>
1. Counting Problems
2. Permutations
3. n-Factorial
4. Permutation of n Things Taken r at a Time
5. Combination of n Things Taken r at a Time
6. The Pascal Triangle
7. The Binomial Theorem and its Proof
8. The Sigma Notation
9. Theorems on the Sigma Notation
10. The Multinomial Theorem

<u>Chapter 3 Introduction to Probability</u>
1. Uses of the Word "Probability"
2. Events, Complementary Events
3. "And" and "Or" Applied to Events
4. Set Symbolism for Events
5. Probability of An Event
6. Equally Likely Simple Events
7. Some Theorems Concerning P(E), Applications
8. Conditional Probability and Formula
9. The Addition and Multiplication Theorems
10. Dependent and Independent Events
11. Tree Diagrams
12. Empirical Definition of Probability

<u>Chapter 4 The Binomial Destribution</u>
1. Repeated Trials
2. Properties of a Repeated Trials Experiment
3. Generalization of the Repeated Trials Experiment
4. Binomial Probabilities for p=1/6
5. Binomial Probability Theorem
6. The Binomial Probabilities--Larger Values of n
7. A Common Error

C. Linear Algebra

Text: Davis, Philip J., <u>The Mathematics of Matrices</u>, Wattham, Massachusetts: Blaisdell Publishing Company, 1965.

<u>Content</u>

<u>Chapter 1 What is a Matrix</u>
1. What is a Matrix
2. The Order of a Matrix
3. Ways of Writing Matrices

4. Tabulating Information as Matrices
5. Equality of Matrices
6. The Transpose of a Matrix

Chapter 2 The Arithmetic of Matrices, Part 1
1. Adding and Subtracting Matrices, Scalar Multiplication
2. Matrix Multiplication, Rules for Matrix Multiplication
3. Matrix Powers and Polynomials
4. The Transpose of a Product
5. Matrix Multiplication at Work
6. Some Practical Problems

Chapter 3 The Arithmetic of Matrices, Part 2
1. The Solution of Linear Equations
2. Elementary Row Transformations
3. Matrix Inversion, the Algebra of Inverses
4. Matrix Inversion by the Gauss-Jordan Scheme
5. Tricks of the Matrix Trade
6. The Sense of Identity
7. Matrix Inversion and the Solution of Systems of Linear Equations

Chapter 4 Linear Transformation of the Plane
1. Functions, Correspondences Transformations, Mappings
2. Transformations of the Plane
3. Matrices as Transformations of the Plane
4. Linear Homogeneous Transformations of the Plane
5. Degenerate Linear Transformations
6. Projections

Chapter 5 Determinants
1. What is a Determinant
2. Determinants, Cofactors, and Inverses
3. An Alternative Approach to Determinants
4. Determinants, Matrices, and Multiplication
5. Additional Algebra of Determinants
6. The Geometry Behind the Multiplication Rule
7. Determinants and Linear Systems of Equations

Chapter 6 Vectors and Inner Products
1. Column Matrices as Vectors
2. Components, Magnitude, and Direction
3. The Inner Product and Angle Geometry
4. Inner Products, Lines, and Half-Planes
5. Converse Sets of Points
6. Vectors and Transformations
7. A Glimpse a n-Dimensional Space.

QUESTIONNAIRE

Please complete the following questionnaire and mail it using the self-addressed envelope. It is not necessary to return the cover letter and the course outlines. Do not write your name on the questionnaire. I am relying on your response.

1. Indicate the department or faculty that best describes your area of instruction or field of interest. Place a check mark (✓) in the blank provided.

_____Engineering

_____Physical Sciences (Chemistry, Physics)

_____Biological Sciences (Botany, Zoology, Genetics)

_____Mathematics

_____Humanities (Art, Drama, Language, Classics)

_____Social Sciences (Anthropology, History, Philosophy, Economics, Political Science, Geography and Sociology)

_____Education (Administration, Secondary Education, Elementary Education, Educational Psychology)

2. Consider the content of the experimental course as outlined and the area of instruction that you indicated in the previous question. On the basis of content alone, which of the three courses do you think is most beneficial to a freshman student who intends to major in your area of instruction? Answer this question by ranking the three experimental courses (one, two, or three) where a rank of one indicates the most appropriate course, a rank of two indicates the next most appropriate course, and a rank of three indicates the least appropriate course. To facilitate ranking, use the numbers 1, 2, or 3. If you have no preference, place a check (✓) in the blank opposite no preference and do not attempt to rank the courses.

COURSE	RANK
An Introduction to Calculus	_____
An Introduction to Sets, Probability, and Hypothesis Testing	_____
Introductory Matrix Algebra	_____
No Preference	_____

3. Answer this question <u>if</u> you stated a preference in question 2.

In your opinion, is it <u>essential</u> that freshman students (who intend to major in your area of instruction) have a knowledge of the course you ranked number one before they enter university? Place a check (✓) in the appropriate blank.

_____Yes

_____No

4. Please make any other comments about these experimental courses that you feel are pertinent. In particular, you may wish to give reasons why you answered the above questions as you did. Also, you may wish to suggest other topics that should be included in the Grade Twelve program.

APPENDIX H

TEACHER QUESTIONNAIRE

Mathematics 31X

The purpose of this questionnaire is to obtain the reactions of the participtaing teachers to the experimental half-courses (calculus, probability and statistics, linear algebra) which are currently being taught as part of the Mathematics 31X course. This questionnaire is not intended as a personal evaluation of the teacher, but rather as an assessment of the half-course itself and how it relates to the students involved.

The questionnaire is divided into five parts as follows:

Part 1 Format of the Textbook
Part 2 The Pupil and the Half-Course
Part 3 Teacher Comments
Part 4 Speculative Questions
Part 5 General Information

Throughout this questionnaire, the phrase "the half-course" refers to the experimental half-course taught during the second half of the Mathematics 31X course. Throughout this questionnaire, the word "trigonometry" refers to the material taught during the first half of the Mathematics 31X course.

<u>Part 1</u>   <u>Format of the Textbook</u>

The following questions pertain to the format of the textbook rather than the content of the textbook. Rate the textbook (excellent, good, fair, poor) on each of the following aspects.

1. The effectiveness of diagrams, graphs, illustrations, etc. ______________
2. The number of exercises provided. ______________
3. The variety of exercises provided (skill, application, comprehension, synthesis) ______________
4. The range of difficulty of exercises provided. ______________
5. The number of sample problems provided. ______________
6. The effectiveness of the written presentations for Mathematics 31 students. ______________
7. The usefulness of the text for individual study by the student. ______________
8. The usefulness of the text as a basis for classroom discussion. ______________
9. The overall appeal of the textbook format. ______________

<u>Part 2</u>   <u>The Pupil and the Half-Course</u>

The following questions are designed to determine how well the half-course relates to the pupil. Please answer the following questions in terms of your own students and in terms of the prescribed half-course. For each of the questions, underline the word(s) that best indicates your judgment.

1. The length of the half-course
   much too long, too long, adequate, too short, much too short.
2. The difficulty of the concepts in the half-course compared to the difficulty of the concepts in trigonometry:
   much more difficult, more difficult, the same, too easy, much too easy.

3. The students' attitude towards the half-course as compared to the students attitude towards trigonometry:
much more favorable, more favorable, the same, less favorable, much less favorable.

4. The mathematical value of the half-course for a grade twelve matriculation student who is mathematically inclined:
very good, good, fair, poor, very poor.

5. Compared to trigonometry, the emphasis placed upon computational skills in the half-course is:
much greater, greater, the same, less, much less.

6. Compared to trigonometry, the emphasis placed upon the understanding of concepts and principles in the half-course is:
much greater, greater, the same, less, much less.

7. Compared to trigonometry, the emphasis placed upon terminology in the half-course is:
much greater, greater, the same, less, much less.

8. Compared to trigonometry, the emphasis placed upon symbolism in the half-course is:
much greater, greater, the same, less, much less.

9. Compared to trigonometry, the concepts presented in the half-course require the students to think:
much more, more, the same, less, much less.

10. Compared to trigonometry, how well do you feel that the half-course is meeting the mathematical needs of your students?
much better, better, about the same, poorer, much poorer.

11. If a student is to benefit by taking the half-course, a "B" or better in Mathematics 20 is:
essential, desirable, unnecessary.

12. If a student is to benefit by taking the half-course, Mathematics 30:
should be taken concurrently, should be a prerequisite, is not required.

13. If a student is to benefit by taking the half-course, trigonometry:
should be taken concurrently, should be a prerequisite, is not required.

Part 3 Teacher Comments

For the questions that follow, please feel free to make any comments or elaborations that you feel are pertinent. If you find that the allotted space is not sufficient, please use the reverse side of the same page. Please discuss these questions on the basis of the material covered in your class.

1. What is your general impression of the half-course?

2. What is your general impression of the prescribed text for the half-course?

3. Did you complete the suggested syllabus? If not, briefly outline the chapters and/or the topics that you covered.

4. Comment on the amount of interest your students showed in the half-course. If your students showed more interest or less interest in the half-course than in trigonometry, to what do you attribute this difference?

5. What method(s) of lesson presentation did you use? (lecture, discussion, student participation, problem-solving sessions, individual study, others) Which of these methods did you find was most effective?

6. Comment on the coherence of the half-course. That is, did you feel that the half-course had a unifying thread running through it or did it tend to be a succession of disjointed topics?

7. In your opinion, was the pupil growth in mathematics sufficient to justify teaching the half-course to your Mathematics 31 students?

8. Considering your own academic training, did you feel well-prepared to teach the half-course? If not, what additional training would you consider adequate?

9. Is there a teacher's guidebook that accompanies the text? If so, did you use it? If you used it, how helpful did you find it?

Part 4 Speculative Questions

In answering the following questions, you need not restrict yourself to your own classroom experience with the half-course. These questions are designed to seek the opinions and judgments of Mathematics 31X teachers as guidance for future revisions in the high school mathematics curriculum. Where applicable, underline the word(s) that best indicate your judgment.

1. The mathematical value of the half-course content for any grade twelve matriculation student:
   very good, good, fair, poor, very poor.

2. The mathematical value of the half-course content for any grade twelve student in the vocational or technical program:
   very good, good, fair, poor, very poor.

3. How well do you think the half-course would accommodate students of varying degrees of ability:
   very well, well, adequately, poorly, very poorly.

4. Suppose that instead of teaching a half-course, a six-week unit was prepared and taught to Mathematics 31 students. The purpose of this unit would be to give the students an overview of the half-course rather than delving into it in depth. Do you think this would be feasible?

5. Do you think that most high school teachers could teach the half-course adequately to Mathematics 31 students?

6. Assuming matriculation-calibre students, at what grade level do you think the half-course could be most profitably taught?

7. Which of the following arrangements would you favor for the Mathematics 31 course? Indicate with an X.
   a) a full year of trigonometry ___
   b) the Mathematics 31X course ___
   c) other (specify) ___

Part 5 General Information

Would you please supply the following information? This information is needed only for recording purposes.

A. Course Information

Name of half-course taught---

Number of students presently in half-course---

Number of minutes of instruction given in the half-course per week---

Please indicate the method used to select students for the half-course. Do you consider your class to be homogeneously or heterogeneously grouped? If your class is homogeneously grouped, what criteria did you use for grouping? (ability, achievement, other)

B. Teacher Information

Number of years of university training---

Degree(s) held---

Is mathematics your major?---

Number of years of teaching experience---

Number of years you have taught high school mathematics---

Explain briefly how you became involved in the Mathematics 31X program.

Using the space below, list by name and number all the university mathematics courses for which you have received credit. If you received credit in the course after 1960, please place an X in the appropriate column.

Course Number	Course Description	After 1960

Did you attend any seminars, non-credit courses, inservice training courses, or informal sessions designed to assist you in teaching the experimental course? If so, did you find these sessions helpful?

Additional Comments

There may be other areas about which you may wish to comment. These might include your personal reaction to teaching the half-course, or areas about which no specific questions have been asked Please feel free to express your comments about any problems you think are significant.

APPENDIX I

SELECTED COMMENTS FROM UNIVERSITY PROFESSORS

A number of selected comments from university professors are contained in this appendix. The information in the parentheses following each comment gives the department or faculty in which the professor instructs as well as the manner in which he ranked the half-courses.

1. The level of instruction in senior courses is limited by the level of mathematics background which can only be rectified by improving the level of mathematics at the secondary school level. These courses could and should have been introduced years ago. The qualifications of teachers, however, made this impossible. (Engineering: Calculus 1, Matrices 2, Probability 3)

2. While a knowledge of the course on calculus is not essential since the students coming to the University now (and in the past) have not had this course and have managed, it is certainly desirable and would give the instructor much greater freedom in expressing himself. I may say that students entering university in Britain have had calculus in their A-level schooling. It will also help bridge the gap between school standards here and the University standards. A narrower "traditional gap" between school and university will, I am sure, benefit the students. (Physical Science: Calculus 1, Matrices 2, Probability 3)

3. All of genetics involves a sense of probability. There are vast differences in the facility with which un-initiated students "pick-up" the notions of sample variance, binomial distributions, etc. I always felt that statistics and probability should come very early in a scientific or liberal arts career because we are so frequently expected to be impressed by specious argument and "testing" advertising. It is a toss-up between linear algebra and calculus but I'd favor calculus just to get it out of the way. (Biological Sciences: Probability 1, Calculus 2, Matrices 3)

4. The preferences which I have indicated are based on the ideal assumption that the courses outlined are being taught by competent instructors at a uniformly good level. Certainly to teach calculus or matrix algebra, the instructor should have at least an honors degree in mathematics. The

student could obtain a completely biased impression of the spirit of modern mathematics from an instructor in matrix algebra who had not undertaken further courses in modern algebra and the theory of linear spaces. (Mathematics: Matrices 1, Probability 2, Calculus 3)

5. None of the experimental courses listed would be essential to a student enrolling in the Humanities. However, each of the experimental half-courses are a good mental discipline which could be beneficial to any student regardless of the field of studies which he or she wishes to follow when entering the University. (Humanities: no preference)

6. A background in logical reasoning is helpful in formulation of plausible hypotheses explaining observed facts in cultural anthropology and archaeology. Matrix analyses is useful in modern statistical approach in physical anthropology. Calculus is not pertinent yet. (Social Sciences: Probability 1, Matrices 2, Calculus 3)

7. I think these courses should be reserved to university level instruction. I think they have no business in the senior year of high school.

   I think the junior high school mathematics courses on which these experimental courses in grade 12 are founded, are producing some unplanned consequences that are highly detrimental to the social structure of our schools and contribute to the discouragement and frustration of many pupils, particularly those from lower socio-economic levels. The enthusiasts for the new mathematics seem to be totally oblivious to the implications of Basil Bernstein's linguistic theory of social learning for their curriculum.

   For an elite group of students the experimental program you describe may have some merits. However, I seriously question whether the aim of secondary education is to prepare a group of elites for the university. The kind of mathematical specialization these courses seem to demand would be better left to the tertiary level of education, not the secondary.

   My field is sociology particularly related to education. Your questionnaire does not permit me to identify myself properly. However, I'm responding as if I were teaching freshman sociology or its equivalent in a Faculty of Education. (Education: Probability 1, Calculus 2, Matrices 3)

APPENDIX J

HIGH SCHOOL AND UNIVERSITY MATRICULATION

EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1967

MATHEMATICS 31X
Calculus

1. If $g(x) = 2|x| - x$, find $g(-\frac{1}{2})$

2. Find the equation of the line passing through the point (1/3, 2) with slope -3.

3. Find, without simplification, the derivatives of:

a) $x^{\frac{1}{2}} - 8x^3 + x^{-2}$

b) $\frac{3x - 1}{x - 2}$

c) $(x^3 - x - 2)^{-2}$

d) $\log(\cos 3x)$

e) 9^x

f) $e^{\arc \cos x}$

g) $\log\sqrt{1 - 2x^2}$

4. Solve for x: $|3x - 2| \leq 1$

5. Find the distance between the points $P_1(-3, -5)$ and $P_2(1, 4)$

6. Evaluate $\cos \frac{2\pi}{6}$

7. If $\lim_{h\to 0} \frac{\sin h}{h} = 1$, find $\lim_{h\to 0} \frac{\sin 4h}{h}$.

8. Find the 'c' of $f'(c)$ of the mean value theorem when $f(x) = x^2 - 3x$, $0 \leq x \leq 2$.

9. Define the absolute value of 'a'.

10.

Use the above diagram to define the slope of a curve at point P.

11. Express in polar coordinate form the position of the point $P_1(1, 1)$ expressed in rectangular form.

12. If $g(x) = \text{arc sin } x$, find $g'(\frac{2}{3})$.

13. Evaluate $y = \text{arc cos}(\cos \frac{5\pi}{4})$, $0 \leq y < 2\pi$.

14. Determine the values of x when the function $f(x) = -x^3 + 2x + 1$ is increasing.

15. Use Newton's Quotient to find the derivative of $x^2 + x$.

16. Using the derivative of $y = \sin x$, find the equation of the tangent line to the curve at $\frac{\pi}{2}$.

17. Find the critical points of the function $\sin x + \cos x$ when $0 \leq x \leq 2\pi$.

18. If $f(x) = x^2 - x + 5$ find $g'(7)$ when $g(y)$ is the inverse function of $f(x)$.

19. Find, without simplification, the derivatives of:

a) $(x^{-\frac{2}{3}} + x^2)(x^3 + \frac{1}{x})$

b) $(x^2 + 1)^3(2x + 5)^2$

c) $\sin\left[\cos(x + 1)\right]$

d) $\log \frac{2x + 1}{x + 3}$

e) $\log(\text{arc sin } x)$

20. Given the function $y = \frac{x + 1}{x^2 + 1}$ find the following:

a) The intersections with the x and y axis

b) The critical points

c) The intervals of

i) increase

ii) decrease

d) The behavior of the function as x becomes very large positively or negatively.

e) Sketch the graph.

21. A box with an open top is to be made with a square base with side x units, height h units, and a constant surface of A square units. Determine the sides of the box if the volume, V cubic units, is to be a maximum.

22. The length of the side of a square, x inches, is increasing at the rate at which the area, A square inches, is increasing when the side is 20 inches long.

HIGH SCHOOL AND UNIVERSITY MATRICULATION

EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1967

MATHEMATICS 31X
Probability

PROBABILITY OF X BEING EQUAL TO

p	0	1	2	3	4	5	6	7	8	9	10
1											1000
0.95							001	010	075	315	599
0.90						002	011	057	194	387	349
0.85					001	008	040	130	276	347	197
0.80				001	006	026	088	201	302	268	107
0.75				003	016	058	146	250	282	188	056
0.70			001	009	037	103	200	267	233	121	028
0.65		001	004	021	069	154	238	252	176	072	013
0.60		002	011	042	111	201	251	215	121	040	006
0.55		004	023	075	160	234	238	166	076	021	003
0.50	001	010	044	117	205	246	205	117	044	010	001
0.45	003	021	076	166	238	234	160	075	023	004	
0.40	006	040	121	215	251	201	111	042	011	002	
0.35	013	072	176	252	238	154	069	021	004	001	
0.30	028	121	233	267	200	103	037	009	001		
0.25	056	188	282	250	146	058	016	003			
0.20	107	268	302	201	088	026	006	001			
0.15	197	347	276	130	040	008	001				
0.10	349	387	194	057	011	002					
0.05	599	315	075	010	001						
0	1000										

Probability Distribution of the Value of X in a Sample of 10 from Populations with $p = 0$, $p = 0.05$, $p = 0.10, \ldots, p = 1$.

Decimal points have been omitted in the body of the table to save space. Each entry is a three-decimal place number. Therefore, each entry in the table should be divided by 1000 to obtain the correct probability.

The empty spaces represent groups of digits "000," each indicating a probability less than .0005.

1. If $P(A) = 0.6$, find $P(A')$.

2. Find the value of each of the following:

   a) $\binom{n}{0}$ b) $(-1)_3$

3. If a die is thrown what is the probability that it shows?

   a) 2 or 6

   b) A number divisible by 7?

   c) A number less than 7?

4. Write in Σ notation the sum of

   $(x + 1)^2 + (x + 2)^2 + (x + 3)^2 + \ldots + (x + n)^2$

5. If $n(A) = 4$ and $n(B) = 6$, find:

   a) $n(A \times B)$

   b) $n(A \times A)$

   c) $n(A \cap B)$ if A adn B are disjoint sets

   d) $n(A \cup B)$ if $n(A \cap B) = 3$.

6. In the expansion of $(a + b)^n$ where n is a positive integer, what is:

   a) the number of terms?

   b) the sum of the coefficients?

   c) the degree of each term?

   d) the 32nd term?

   e) the term that contains a^{n-6}?

7. $S = 0_1, 0_2, 0_3, 0_4, 0_5,$. If $P(0_1) = \frac{1}{4}$, $P(0_2) = \frac{1}{6}$,

   $P(0_3) = \frac{1}{12}$ and $P(0_4) = \frac{1}{3}$, find $P(0_5)$.

8. If $P(A) = 0.4$, $P(B) = 0.5$, and $P(A\ B) = 0.2$, find:

   a) $P(A|B)$

   b) $P(A'|B')$

   c) Are events A and B dependent or independent? Give reason for your answer.

9. Two dice labeled I and II are thrown.

   a) Indicate an appropriate sample space.

   b) What is the probability that the sum of the showings is 4?

   c) What is the probability that the first die shows 1?

   d) What is the probability that if the first die shows 1 the sum of the showings is 4?

10. Six boys, John, Joe, Tom, Bill, George and Frank, wish to form a committee of 3.

    a) How many different committees can be formed?

    b) How many can be formed if John and Joe will not serve on the same committee?

    c) Find the probability of having a committee composed of John, Bill and Frank.

    d) Find the probability of having a committee with at least one of John, Bill and Frank serving on it.

11. During an epidemic it is estimated that the probability of a person having influenza is $\frac{1}{3}$. In a group of 5 people what is the probability that:

    a) no one will have influenza?

    b) at least two will have influenza?

12. A sample of 10 with $X \leq 4$ will lead to the rejection of the hypothesis $p = .70$. What is the probability of rejecting the hypothesis if it is true?

13. A radio station claims that at least 40% of the radios that are turned on are tuned to it. You take a random sample of 10 cases and decide to reject the station's claim if 0, 1, or 2 are tuned to the station. Complete the following:

POPULATION

H

EXPERIMENT

S

NATURE OF TEST

DECISION RULE

CRITICAL REGION OF S

GRAPH OF S

α

Do you think your decision rule is a fair one? Why?

14. On the basis of a season's play it is calculuated that Chicago will beat Toronto 2 games in every 3 and Toronto will beat Chicago 1 game in every 3. What is the probability that Toronto in a seven game series will win:

a) at least two of the first three games?

b) the series in exactly six games?

HIGH SCHOOL AND UNIVERSITY MATRICULATION

EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1967

MATHEMATICS 31X
Linear Algebra

1. Find the product:

$$\begin{pmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & -2 & -1 \end{pmatrix} \begin{pmatrix} 3 & -1 & -3 \\ 4 & -2 & 2 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{pmatrix} =$$

2. Given:

$$\begin{cases} a = x + 2y \\ b = x + 3y + z \\ c = -x + z \end{cases} \qquad \begin{cases} w = a + 2b \\ u = a + b + c \\ y = a - b + 2c \end{cases}$$

Find w, u, v, in terms of x, y, z.

3. If A and B are square matrices of the same order, is $(A + B)^2 = A^2 + 2AB + B^2$? Explain the reason for your answer.

4. Is the following system determined, overdetermined or undetermined? Support your answer.

$$2x_1 + x_2 = 7$$

$$x_1 + 2x_2 = 5$$

$$3x_1 - 2x_2 = 7$$

5. If A and B are any two matrices that are conformable for multiplication in the order AB, explain why $B'A'$ would be conformable for the multiplication in that order.

6. Express the t's directly in terms of s's by means of matrix multiplication if:

$$\begin{cases} r_1 = s_1 + 2s_2 \\ r_2 = 2s_1 - s_2 \\ r_3 = 2s_1 + 3s_2 \end{cases} \qquad \begin{cases} q_1 = r_1 - r_2 \\ q_2 = 2r_1 - 3r_2 \\ q_3 = r_1 + 3r_2 \end{cases}$$

$$\begin{cases} t_1 = -2q_1 + q_2 - q_3 \\ t_2 = 3q_1 - q_2 + 2q_3 \end{cases}$$

7. Solve for x, y and z by means of the Gauss-Jordan elimination method.

$$\begin{aligned} x + y + z + w &= 4 \\ x \qquad\quad - w &= -6 \\ x \quad - z + w &= -2 \\ x - y + z + w &= 0 \end{aligned}$$

8. Explain the effect of multiplying on the left by:

a) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 0 & 1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

9. Find B if

$$(B')^{-1} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

10. Find the inverse (if one exists):

a)
$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

b)
$$B = \begin{pmatrix} 2 & -1 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

11. Given:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} -2 & 6 \\ 1 & -3 \end{pmatrix}$$

find the product AB and explain the result.

12. Determine which matrix or matrices are singular and which are non-singular. Find an inverse for each of those that have one.

$$\begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 9 \\ 0 & 8 \end{pmatrix}$$

13. Interpret in terms of transformations:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 7 \\ 0 & 1 \end{pmatrix}$$

14. Determine if the following matrix is orthogonal:

$$\begin{matrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{matrix}$$

15. Transform the point (2, -4) by a rotation through 60° about the origin.

16. Express in matrix form the one transformation that would result when a figure undergoes the succession of transformations in the following order.

a) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

b) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

c) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

17. Supply the missing members in the following identities:

a) $A^{-1}B^{-1}C^{-1} =$

b) $(A + B)' =$

c) $(A + B)C =$

d) $(aA)^{-1} =$

18. The final record of the basketball city league is given in the matrix in which the element a_{ij} is the number of games the i^{th} team won from the j^{th} team.

Team	A	B	C	D
A	0	8	12	7
B	8	0	10	6
C	4	6	0	14
D	9	10	2	0

a) Interpret the first row sum.

b) Interpret the first column sum.

c) If $i \neq j$, then $a_{ij} + a_{ji}$ always equals 16. Explain.

d) How many games did each team play?